UNIVERSITY OF DELHI

CNC-II/093/1(26)/2023-24/194

Dated: 14.09.2023

NOTIFICATION

Sub: Amendment to Ordinance V

[E.C Resolution No. 14/ (14-1-7/) and 27-1-2/ dated 09.06.2023 and 25.08.2023 respectively]

Following addition be made to Appendix-II-A to the Ordinance V (2-A) of the Ordinances of the University;

Add the following:

Syllabi of Semester-IV, V and VI of the following departments under Faculty of Mathematical Sciences based on Under Graduate Curriculum Framework -2022 implemented from the Academic Year 2022-23.

FACULTY OF MATHEMATICAL SCIENCES

1. Department of Mathematics
2. Department of Statistics
3. Department of Operational Research
4. Department of Computer Science

DEPARTMENT OF MATHEMATICS

Category-I

B.Sc. (Hons.) Mathematics, Semester-IV

DISCIPLINE SPECIFIC CORE COURSE – 10: SEQUENCES AND SERIES OF FUNCTIONS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Sequences and Series of Functions	4	3	1	0	Class XII pass with Mathematics	DSC-2: Real Analysis DSC-5: Calculus DSC-8: Riemann Integration

Learning Objectives: The objective of the course is to introduce:

- The sequences and series of real-valued functions as a generalization to the sequences and series of real numbers.
- The situations under which the process of convergence of a sequence and series of real-valued functions may commute with the processes of calculus while taking differentiation, or integration.
- An important class of series functions (i.e., power series), and the elementary functions-exponential, logarithmic and trigonometric.

Learning Outcomes: This course will enable the students to:

- Learn about Cauchy criterion for uniform convergence and Weierstrass *M*-test for uniform convergence of series of real-valued functions.
- Know about the constraints for the inter-changeability of differentiation, and integration with infinite sum of a series of functions.
- Handle the convergence of power series and properties of the limit function, including differentiation and integration of power series.
- Appreciate utility of polynomials in the space of continuous functions.

SYLLABUS OF DSC-10

UNIT – I: Sequences of Functions (18 hours)

Pointwise and uniform convergence of sequence of functions, The uniform norm, Cauchy criterion for uniform convergence, Continuity of the limit function of a sequence of functions, Interchange of the limit and derivative, and the interchange of the limit and integral of a sequence of functions, Bounded convergence theorem.

UNIT – II: Series of Functions (12 hours)

Pointwise and uniform convergence of series of functions, Theorems on the continuity, differentiability and integrability of the sum function of a series of functions, Cauchy criterion and the Weierstrass *M*-test for uniform convergence.

UNIT – III: Power Series (15 hours)

Definition of a power series, Radius of convergence, Absolute convergence (Cauchy-Hadamard theorem), Differentiation and integration of power series, Abel's theorem, Weierstrass's approximation theorem; The exponential, logarithmic and trigonometric functions: Definitions and their basic properties.

Essential Readings

1. Bartle, Robert G., & Sherbert, Donald R. (2011). Introduction to Real Analysis (4th ed.). Wiley India Edition. Indian Reprint.
2. Ross, Kenneth A. (2013). Elementary Analysis: The Theory of Calculus (2nd ed.). Undergraduate Texts in Mathematics, Springer. Indian Reprint.

Suggestive Readings

- Bilodeau, Gerald G., Thie, Paul R., & Keough, G. E. (2010). An Introduction to Analysis (2nd ed.). Jones and Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.
- Denlinger, Charles G. (2011). Elements of Real Analysis. Jones and Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

DISCIPLINE SPECIFIC CORE COURSE – 11: MULTIVARIATE CALCULUS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Multivariate Calculus	4	3	1	0	Class XII pass with Mathematics	DSC-2: Real Analysis DSC-5: Calculus DSC-8: Riemann Integration

Learning Objectives: The primary objective of this course is to introduce:

- The extension of the studies of single variable differential and integral calculus to functions of two or more independent variables.
- The geometry and visualisation of curves and surfaces in two dimensions (plane) and three dimensions (space).
- The techniques of integration to functions of two and three independent variables.
- The applications of multivariate calculus tools to physics, economics, optimization etc.

Learning Outcomes: This course will enable the students to:

- Learn the conceptual variations when advancing in calculus from one variable to multivariable discussion.
- Understand the maximization and minimization of multivariable functions subject to the given constraints on variables.
- Learn about inter-relationship amongst the line integral, double, and triple integral formulations.
- Familiarize with Green's, Stokes' and Gauss divergence theorems, and learn applications.

SYLLABUS OF DSC-11

UNIT – I: Calculus of Functions of Several Variables (18 hours)

Basic concepts, Limits and continuity, Partial derivatives, Tangent planes, Total differential, Differentiability, Chain rules, Directional derivatives and the gradient, Extrema of functions of two variables, Method of Lagrange multipliers with one constraint.

UNIT – II: Double and Triple Integrals (15 hours)

Double integration over rectangular and nonrectangular regions, Double integrals in polar co-ordinates, Triple integrals over a parallelopiped and solid regions, Volume by triple integrals, Triple integration in cylindrical and spherical coordinates, Change of variables in double and triple integrals.

UNIT – III: Green's, Stokes' and Gauss Divergence Theorem (12 hours)

Vector field, Divergence and curl, Line integrals and applications to mass and work, Fundamental theorem for line integrals, Conservative vector fields, Green's theorem, Area as a line integral, Surface integrals, Stokes' theorem, Gauss divergence theorem.

Essential Reading

1. Strauss, Monty J., Bradley, Gerald L., & Smith, Karl J. (2007). Calculus (3rd ed.). Dorling Kindersley (India) Pvt. Ltd. Pearson Education. Indian Reprint.

Suggestive Reading

- Marsden, J. E., Tromba, A., & Weinstein, A. (2004). Basic Multivariable Calculus. Springer (SIE). Indian Reprint.

DISCIPLINE SPECIFIC CORE COURSE – 12: NUMERICAL ANALYSIS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Numerical Analysis	4	3	0	1	Class XII pass with Mathematics	DSC-2: Real Analysis DSC-5: Calculus

Learning Objectives: The main objective of this course is to introduce:

- Various computational techniques to find approximate value for possible root(s) of algebraic and non-algebraic equations.
- Methods to solve system of linear equations and ordinary differential equations.
- The use of computer algebra system (CAS) by which the numerical problems can be solved both numerically and analytically, and to enhance the problem-solving skills.

Learning Outcomes: This course will enable the students to:

- Learn some numerical methods to find the zeroes of nonlinear functions of a single variable, up to a certain given level of precision.
- Learn Gauss−Jacobi, Gauss−Seidel methods to solve system of linear equations.
- Get aware of using interpolation techniques, for example in finding values of a tabulated function at points which are not part of the table.
- Learn finding numerical solutions of difference equations which are obtained converting differential equations using techniques from calculus.

SYLLABUS OF DSC-12

UNIT – I: Methods for Solving Algebraic and Transcendental Equations (12 hours)

Rate and order of convergence; Bisection method, Method of false position, Fixed point iteration method, Newton's method, and Secant method, their order of convergence and convergence analysis.

UNIT – II: Techniques to Solve Linear Systems and Interpolation (15 hours)

LU decomposition and its applications; Iterative methods: Gauss−Jacobi, Gauss−Seidel methods; Lagrange and Newton interpolation, Piecewise linear interpolation.

UNIT – III: Numerical Differentiation and Integration (18 hours)

First and higher order approximation for the first derivative, Approximation for the second derivative; Numerical integration by closed Newton−Cotes formulae: Trapezoidal rule, Simpson's rule and its error analysis; Euler's method to solve ODE's, Modified Euler method, Runge−Kutta Method (fourth-order).

Essential Reading

1. Bradie, Brian. (2006). A Friendly Introduction to Numerical Analysis. Pearson Education India. Dorling Kindersley (India) Pvt. Ltd. Third impression 2011.

Suggestive Readings

- Gerald, Curtis F., & Wheatley, Patrick O. (2007). Applied Numerical Analysis (7th ed.). Pearson Education. India.
- Jain, M. K., Iyengar, S. R. K., & Jain, R. K. (2012). Numerical Methods for Scientific and Engineering Computation. (6th ed.). New Age International Publisher, India, 2016.

Note: Non programmable scientific calculator may be allowed in the University examination.

Practical (30 hours)- Practical / Lab work to be performed in Computer Lab: Use of computer algebra system (CAS) software: Python/SageMath/Mathematica/MATLAB/Maple/Maxima/ Scilab etc., for developing the following numerical programs:

1. Bisection method.

2. Newton-Raphson method.
3. Secant method.
4. LU decomposition method.
5. Gauss−Jacobi method.
6. Gauss−Seidel method.
7. Lagrange interpolation.
8. Newton interpolation.
9. Trapezoidal rule.
10. Simpson's rule.
11. Euler’s method.
12. Runge−Kutta Method (fourth-order).

Note: Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

B.Sc. (Hons) Mathematics, Semester-IV, DSE-Courses

DISCIPLINE SPECIFIC ELECTIVE COURSE – 2(i): BIOMATHEMATICS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Biomathematics	4	3	1	0	Class XII pass with Mathematics	DSC-6: Ordinary Differential Equations

Learning Objectives: The main objective of this course is to:

- Develop and analyse the models of the biological phenomenon with emphasis on population growth and predator-prey models.
- Interpret first-order autonomous systems of nonlinear differential equations using the Poincaré phase plane.
- Apply the basic concepts of probability to understand molecular evolution and genetics.

Learning Outcomes: This course will enable the students to:

- To learn and appreciate study of long-term behavior arising naturally in study of mathematical models and their impact on society at large.
- To understand spread of epidemic technically through various models and impact of recurrence phenomena.
- Learn what properties like Chaos and bifurcation means through various examples and their impact in Bio-Sciences.

SYLLABUS OF DSE-2(i)

UNIT – I: Mathematical Modeling for Biological Processes (15 hours)

Formulation a model through data, A continuous population growth model, Long-term behavior and equilibrium states, The Verhulst model for discrete population growth, Administration of drugs, Differential equation of chemical process and predator-prey model (Function response: Types I, II and III).

UNIT – II: Epidemic Model: Formulation and Analysis (15 hours)

Introduction to infectious disease, The SIS, SIR and SEIR models of the spread of an epidemic, Analyzing equilibrium states, Phase plane analysis, Stability of equilibrium points, Classifying the equilibrium state; Local stability, Limit cycles, Poincaré-Bendixson theorem.

UNIT – III: Bifurcation, Chaos and Modeling Molecular Evolution (15 hours)

Bifurcation, Bifurcation of a limit cycle, Discrete bifurcation and period-doubling, Chaos, Stability of limit cycles, Introduction of the Poincaré plane; Modeling molecular evolution: Matrix models of base substitutions for DNA sequences, Jukes-Cantor and Kimura models, Phylogenetic distances.

Essential Readings

1. Robeva, Raina S., et al. (2008). An Invitation to Biomathematics. Academic press.
2. Jones, D. S., Plank, M. J., & Sleeman, B. D. (2009). Differential Equations and Mathematical Biology (2nd ed.). CRC Press, Taylor & Francis Group.
3. Allman, Elizabeth S., & Rhodes, John A. (2004). Mathematical Models in Biology: An Introduction. Cambridge University Press.

Suggestive Readings

- Linda J. S. Allen (2007). An Introduction to Mathematical Biology. Pearson Education.
- Murray, J. D. (2002). Mathematical Biology: An Introduction (3rd ed.). Springer.
- Shonkwiler, Ronald W., & Herod, James. (2009). Mathematical Biology: An Introduction with Maple and MATLAB (2nd ed.). Springer.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 2(ii): MATHEMATICAL MODELING

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Mathematical Modeling	4	3	0	1	Class XII pass with Mathematics	DSC-6: Ordinary Differential Equations

Learning Objectives: Primary objective of this course is to introduce:

- Mathematical modeling as the representation of a system by a set of mathematical relations or equations.
- Mathematical epidemiological models susceptible-infectious-recovered (SIR) and its variant SEIR (S-Exposed-IR) for the spread of diseases.
- Monte Carlo simulation techniques, and simplex method for solving linear programming problems.

Learning Outcomes: This course will enable the students to:

- Understand the methodology of solving SIR models for disease spread.
- Learn significance of dieting model that provides important insights and guides to a biomedical issue that is of interest to the general public.
- Understand nonlinear systems and phenomena with stability analysis ranges from phase plane analysis to ecological and mechanical systems.

- Use Monte Carlo simulation technique to approximate area under a given curve, and volume under a given surface.

SYLLABUS OF DSE-2(ii)

UNIT – I: Mathematical Epidemiological and Dieting Models (15 hours)
Modeling concepts and examples, Scaling of variables, and approximations of functions; SIR and SEIR models for disease spread: Methodology, Standard and solvable SIR models, Basic reproduction number; Dieting model with analysis and approximate solutions.

UNIT – II: Modeling with Nonlinear Systems and Phenomena (15 hours)
Stability and the phase plane, Almost linear systems; Ecological models: Predators and competitors, Critical points, Oscillating populations, Survival of single species, Peaceful coexistence of two species, Interaction of logistic populations, Wildlife conservation preserve; Nonlinear mechanical systems: Hard and soft spring oscillations, Damped nonlinear vibrations.

UNIT – III: Simulation and Optimization Modeling (15 hours)
Monte Carlo simulating deterministic, and probabilistic behavior, Generating random numbers; Linear programming model: Geometric and algebraic solutions, Simplex method and its tableau format, Sensitivity analysis.

Essential Readings

1. Mickens, Ronald E. (2022). Mathematical Modelling with Differential Equations. CRC Press, Taylor & Francis Group.
2. Edwards, C. Henry, Penney, David E., & Calvis, David T. (2023). Differential Equations and Boundary Value Problems: Computing and Modeling (6th ed.). Pearson.
3. Giordano, Frank R., Fox, William P., & Horton, Steven B. (2014). A First Course in Mathematical Modeling (5th ed.). Brooks/Cole, Cengage Learning India Pvt. Ltd.

Suggestive Readings

- Barnes, Belinda & Fulford, Glenn R. (2015). Mathematical Modeling with Case Studies, Using Maple and MATLAB (3rd ed.). CRC Press. Taylor & Francis Group.
- Ross, Shepley L. (2014). Differential Equations (3rd ed.). Wiley India Pvt. Ltd.
- Simmons, George F. (2017). Differential Equations with Applications and Historical Notes (3rd ed.). CRC Press. Taylor & Francis Group.

Practical (30 hours)- Practical work to be performed in Computer Lab: Modeling of the following problems using: R/Python/SageMath/Mathematica/MATLAB/Maxima/Scilab etc.

1. a) Simulation of SIR model and its variants using some initial parameter values, and finding basic reproduction number for analysis.
   b) Analysis of the dieting process, which includes both body-mass loss and gain.
2. Nonlinear Systems and Phenomena.
   a) Plot phase plane portraits and solutions of first-order equations.
   b) Obtain interesting and complicated phase portraits for almost linear systems.

c) Discuss large wildlife conservation preserve model and obtain (i) The period of oscillation of the rabbit and fox populations, (ii) The maximum and minimum numbers of rabbits and foxes.

d) Discuss the Rayleigh and van der Pol models.

3. (i) Random number generation and then use it for the following:
   a) Simulate area under a given curve.
   b) Simulate volume under a given surface.

   (ii) [2] Chapter 7 (Projects 7.4 and 7.5).

DISCIPLINE SPECIFIC ELECTIVE COURSE – 2(iii): MECHANICS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Mechanics	4	3	1	0	Class XII pass with Mathematics	DSC-5: Calculus DSC-6: Ordinary Differential Equations

Learning Objectives: The main objective of this course is to:

- Starting Newtonian laws, learning various technical notions which explains various states of motion under given forces.
- Deals with the kinematics and kinetics of the rectilinear and planar motions of a particle including constrained oscillatory motions of particles, projectiles, and planetary orbits.
- Understand hydrostatic pressure and thrust on plane surfaces.

Learning Outcomes: This course will enable the students to:

- Understand necessary conditions for the equilibrium of particles acted upon by various forces and learn the principle of virtual work for a system of coplanar forces.
- Apply the concepts of center of gravity, laws of static and kinetic friction.
- Learn that a particle moving under a central force describes a plane curve and know the Kepler's laws of the planetary motions.
- Evaluate the hydrostatic pressure at any given depth in a heavy homogeneous liquid at rest under gravity.

SYLLABUS OF DSE-2(iii)

UNIT – I: Statics (15 hours)

Fundamental laws of Newtonian mechanics, Law of parallelogram of forces, Equilibrium of a particle, Lamy's theorem, Equilibrium of a system of particles, External and internal forces, Couples, Reduction of a plane force system, Work, Principle of virtual work, Potential energy and conservative field, Mass centers, Centers of gravity, Friction.

UNIT – II: Dynamics **(18 hours)**

Kinemetics of a particle, Motion of a particle, Motion of a system, Principle of linear momentum, Motion of mass center, Principle of angular momentum, Motion relative to mass center, Principle of energy, D'Alembert's principle; Moving frames of reference, Frames of reference with uniform translational velocity, Frames of reference with constant angular velocity; Applications in plane dynamics- Motion of a projectile, Harmonic oscillators, General motion under central forces, Planetary orbits.

UNIT – III: Hydrostatics **(12 hours)**

Shearing stress, Pressure, Perfect fluid, Pressure at a point in a fluid, Transmissibility of liquid pressure, Compression, Specific gravity, Pressure of heavy fluid- Pressure at all points in a horizontal plane, Surface of equal density; Thrust on plane surfaces.

Essential Readings

1. Synge, J. L., & Griffith, B. A. (2017). Principles of Mechanics (3rd ed.). McGraw-Hill Education. Indian Reprint.
2. Ramsey, A. S. (2017). Hydrostatics. Cambridge University Press. Indian Reprint.

Suggestive Readings

- Roberts, A. P. (2003). Statics and Dynamics with Background Mathematics. Cambridge University Press.
- Ramsey, A. S. (1985). Statics (2nd ed.). Cambridge University Press.

B.A. (Prog.) Semester-IV with Mathematics as Major
Category-II

DISCIPLINE SPECIFIC CORE COURSE (DSC-4): INTRODUCTION TO GRAPH THEORY

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Introduction to Graph Theory	4	3	1	0	Class XII pass with Mathematics	NIL

Learning Objectives: The primary objective of this course is to introduce:
- Problem-solving techniques using various concepts of graph theory.
- Various properties like planarity and chromaticity of graphs.
- Several applications of these concepts in solving practical problems.

Learning Outcomes: This course will enable the students to:
- Good familiarity with all initial notions of graph theory and related results and seeing them used for some real-life problems.
- Learning notion of trees and their enormous usefulness in various problems.
- Learning various algorithms and their applicability.
- Studying planar graphs, Euler theorem associated to such graphs and some useful applications like coloring of graphs.

SYLLABUS OF DSC-4

UNIT-I: Graphs, Types of Graphs and Basic Properties (12 hours)
Graphs and their representation, Pseudographs, Subgraphs, Degree sequence, Euler's theorem, Isomorphism of graphs, Paths and circuits, Connected graphs, Euler trails and circuits, Hamiltonian paths and cycles, Adjacency matrix, Weighted graphs, Travelling salesman problem, Dijkstra's algorithm.

UNIT-II: Directed Graphs and Applications, Trees (18 hours)
The Chinese postman problem; Digraphs, Bellman-Ford algorithm, Tournaments, Directed network, Scheduling problem; Trees and their properties, Spanning trees, Kruskal's algorithm, Prim's algorithm, Acyclic digraphs and Bellman's algorithm.

UNIT-III: Planar Graphs, Graph Coloring and Network Flows (15 hours)
Planar graphs, Euler's formula, Kuratowski theorem, Graph coloring, Applications of graph

coloring, Circuit testing and facilities design, Flows and cuts, Max flow-min cut theorem, Matchings, Hall's theorem.

Essential Reading

1. Goodaire, Edgar G., & Parmenter, Michael M. (2011). Discrete Mathematics with Graph Theory (3rd ed.). Pearson Education Pvt. Ltd. Indian Reprint.

Suggestive Readings

- Bondy, J. A. & Murty, U.S.R. (2008), Graph Theory with Applications. Springer.
- Chartrand, Gary, & Zhang, P. (2012). A First Course in Graph Theory. Dover Publications.
- Diestel, R. (1997). Graph Theory (Graduate Texts in Mathematics). Springer Verlag.
- West, Douglas B. (2001). Introduction to graph theory (2nd ed.). Pearson India.

DISCIPLINE SPECIFIC CORE COURSE – 4 (Discipline A-4): ABSTRACT ALGEBRA

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Abstract Algebra	4	3	1	0	Class XII pass with Mathematics	NIL

Learning Objectives: The primary objective of the course is to introduce:

- Modular arithmetic, fundamental theory of groups, rings, integral domains, and fields.
- Symmetry group of a plane figure, and basic concepts of cyclic groups.
- Cosets of a group and its properties, Lagrange's theorem, and quotient groups.

Learning Outcomes: This course will enable the students to:

- Appreciate ample types of groups present around us which explains our surrounding better, and classify them as abelian, cyclic and permutation groups.
- Explain the significance of the notion of cosets, normal subgroups and homomorphisms.
- Understand the fundamental concepts of rings, subrings, fields, ideals, and factor rings.

SYLLABUS OF DISCIPLINE A-4

UNIT-I: Introduction to Groups (12 hours)

Modular arithmetic; Definition and examples of groups, Elementary properties of groups, Order of a group and order of an element of a group; Subgroups and its examples, Subgroup tests; Center of a group and centralizer of an element of a group.

UNIT-II: Cyclic Groups, Permutation Groups and Lagrange's Theorem (18 hours)

Cyclic groups and its properties, Generators of a cyclic group; Group of symmetries; Permutation groups, Cyclic decomposition of permutations and its properties, Even and odd permutations and the alternating group; Cosets and Lagrange's theorem; Definition and examples of normal subgroups, Quotient groups; Group homomorphisms and properties.

UNIT-III: Rings, Integral Domains and Fields (15 hours)

Definition, examples and properties of rings, subrings, integral domains, fields, ideals and factor rings; Characteristic of a ring; Ring homomorphisms and properties.

Essential Reading

1. Gallian, Joseph. A. (2017). Contemporary Abstract Algebra (9th ed.). Cengage Learning India Private Limited, Delhi. Indian Reprint (2021).

Suggestive Reading

- Beachy, John A., & Blair, William D. (2006). Abstract Algebra (3rd ed.). Waveland Press.

B.Sc. (Prog.)/ BA (Prog.) Semester-IV with Mathematics as non-Major

Category-III

DISCIPLINE SPECIFIC CORE COURSE – 4 (Discipline A-4): ABSTRACT ALGEBRA

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Abstract Algebra	4	3	1	0	Class XII pass with Mathematics	NIL

Learning Objectives: The primary objective of the course is to introduce:

- Modular arithmetic, fundamental theory of groups, rings, integral domains, and fields.
- Symmetry group of a plane figure, and basic concepts of cyclic groups.
- Cosets of a group and its properties, Lagrange's theorem, and quotient groups.

Learning Outcomes: This course will enable the students to:

- Appreciate ample types of groups present around us which explains our surrounding better, and classify them as abelian, cyclic and permutation groups.
- Explain the significance of the notion of cosets, normal subgroups and homomorphisms.
- Understand the fundamental concepts of rings, subrings, fields, ideals, and factor rings.

SYLLABUS OF DISCIPLINE A-4

UNIT-I: Introduction to Groups (12 hours)

Modular arithmetic; Definition and examples of groups, Elementary properties of groups, Order of a group and order of an element of a group; Subgroups and its examples, Subgroup tests; Center of a group and centralizer of an element of a group.

UNIT-II: Cyclic Groups, Permutation Groups and Lagrange's Theorem (18 hours)

Cyclic groups and its properties, Generators of a cyclic group; Group of symmetries; Permutation groups, Cyclic decomposition of permutations and its properties, Even and odd permutations and the alternating group; Cosets and Lagrange's theorem; Definition and examples of normal subgroups, Quotient groups; Group homomorphisms and properties.

UNIT-III: Rings, Integral Domains and Fields (15 hours)

Definition, examples and properties of rings, subrings, integral domains, fields, ideals and factor rings; Characteristic of a ring; Ring homomorphisms and properties.

Essential Reading

1. Gallian, Joseph. A. (2017). Contemporary Abstract Algebra (9th ed.). Cengage Learning India Private Limited, Delhi. Indian Reprint (2021).

Suggestive Reading

- Beachy, John A., & Blair, William D. (2006). Abstract Algebra (3rd ed.). Waveland Press.

B.Sc. (Physical Sciences/Mathematical Sciences) Semester-IV with Mathematics as one of the Core Discipline

Category-III

DISCIPLINE SPECIFIC CORE COURSE – 4 (Discipline A-4): ABSTRACT ALGEBRA

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Abstract Algebra	4	3	1	0	Class XII pass with Mathematics	NIL

Learning Objectives: The primary objective of the course is to introduce:

- Modular arithmetic, fundamental theory of groups, rings, integral domains, and fields.
- Symmetry group of a plane figure, and basic concepts of cyclic groups.
- Cosets of a group and its properties, Lagrange's theorem, and quotient groups.

Learning Outcomes: This course will enable the students to:

- Appreciate ample types of groups present around us which explains our surrounding better, and classify them as abelian, cyclic and permutation groups.
- Explain the significance of the notion of cosets, normal subgroups and homomorphisms.
- Understand the fundamental concepts of rings, subrings, fields, ideals, and factor rings.

SYLLABUS OF DISCIPLINE A-4

UNIT-I: Introduction to Groups (12 hours)

Modular arithmetic; Definition and examples of groups, Elementary properties of groups, Order of a group and order of an element of a group; Subgroups and its examples, Subgroup tests; Center of a group and centralizer of an element of a group.

UNIT-II: Cyclic Groups, Permutation Groups and Lagrange's Theorem (18 hours)

Cyclic groups and its properties, Generators of a cyclic group; Group of symmetries; Permutation groups, Cyclic decomposition of permutations and its properties, Even and odd permutations and the alternating group; Cosets and Lagrange's theorem; Definition and examples of normal subgroups, Quotient groups; Group homomorphisms and properties.

UNIT-III: Rings, Integral Domains and Fields (15 hours)

Definition, examples and properties of rings, subrings, integral domains, fields, ideals and factor rings; Characteristic of a ring; Ring homomorphisms and properties.

Essential Reading

1. Gallian, Joseph. A. (2017). Contemporary Abstract Algebra (9th ed.). Cengage Learning India Private Limited, Delhi. Indian Reprint (2021).

Suggestive Reading

- Beachy, John A., & Blair, William D. (2006). Abstract Algebra (3rd ed.). Waveland Press.

DSE Courses of B.Sc. (Physical Sciences/Mathematical Sciences) Semester-IV

Category-III

DISCIPLINE SPECIFIC ELECTIVE COURSE – 2(i): ELEMENTS OF DISCRETE MATHEMATICS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Elements of Discrete Mathematics	4	3	1	0	Class XII pass with Mathematics	NIL

Learning Objectives: Students are introducing to:

- Order (or partial order) and related properties.
- Notion of a lattice which is also a step towards abstract algebra.
- Concept of Boolean algebra and its applications to minimizing a Boolean polynomial and switching circuits, which has further applications in computer science.

Learning Outcomes: This course will enable the students to:

- Understand the basic concepts of sets, relations, functions, and induction.
- Understand mathematical logic and logical operations to various fields.
- Understand the notion of order and maps between partially ordered sets.
- Minimize a Boolean polynomial and apply Boolean algebra techniques to decode switching circuits.

SYLLABUS OF DSE-2(i)

UNIT-I: Sets, Relations and Functions (18 hours)

Sets, Propositions and logical operations, Conditional statements, Mathematical induction, Relations and equivalence relation, Equivalence classes, Partial order relation, Partially ordered set, Hasse diagrams, Chain, Maximal and minimal elements, least and greatest elements, Least upper bound, Greatest lower bound, Zorn's lemma, Functions and bijective functions, Functions between POSETS, Order isomorphism.

UNIT-II: Lattices (12 hours)

Lattice as a POSET, Lattice as an algebra and their equivalence, Bounded lattices, Sublattices, Interval in a lattice, Products and homomorphism of lattices, Isomorphism of lattices; Distributive, Complemented, Partition and pentagonal lattices.

UNIT-III: Boolean Algebra and Switching Circuits (15 hours)

Boolean algebra, De Morgan's laws, Boolean expressions, Truth tables, Logic diagrams, Boolean functions, Disjunctive normal forms (as join of meets), Minimal forms of Boolean polynomials, Quine Mc-Cluskey method, Karnaugh maps, Switching circuits, Applications of switching circuits.

Essential Readings

1. Rudolf Lidl, & Gunter Pilz (2004). Applied Abstract Algebra (2nd ed.). Undergraduate text in Mathematics, Springer (SIE), Indian Reprint.
2. Bernard Kolman, Robert C. Busby, & Sharon Cutler Ross (2009). Discrete Mathematical Structures (6th ed.). Pearson education Inc., Indian reprint.

Suggestive Reading

- Rosen, Kenneth H. (2017). Discrete Mathematics and its applications with combinatorics and Graph Theory (7th ed.). McGraw Hill Education.

DISCIPLINE SPECIFIC ELECTIVE COURSE-2(ii): INTRODUCTION TO GRAPH THEORY

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Introduction to Graph Theory	4	3	1	0	Class XII pass with Mathematics	NIL

Learning Objectives: The primary objective of this course is to introduce:

- Problem-solving techniques using various concepts of graph theory.
- Various properties like planarity and chromaticity of graphs.
- Several applications of these concepts in solving practical problems.

Learning Outcomes: This course will enable the students to:

- Good familiarity with all initial notions of graph theory and related results and seeing them used for some real-life problems.
- Learning notion of trees and their enormous usefulness in various problems.
- Learning various algorithms and their applicability.
- Studying planar graphs, Euler theorem associated to such graphs and some useful applications like coloring of graphs.

SYLLABUS OF DSE-2(ii)

UNIT-I: Graphs, Types of Graphs and Basic Properties (12 hours)

Graphs and their representation, Pseudographs, Subgraphs, Degree sequence, Euler's theorem, Isomorphism of graphs, Paths and circuits, Connected graphs, Euler trails and circuits, Hamiltonian paths and cycles, Adjacency matrix, Weighted graphs, Travelling salesman problem, Dijkstra's algorithm.

UNIT-II: Directed Graphs and Applications, Trees (18 hours)

The Chinese postman problem; Digraphs, Bellman-Ford algorithm, Tournaments, Directed network, Scheduling problem; Trees and their properties, Spanning trees, Kruskal's algorithm, Prim's algorithm, Acyclic digraphs and Bellman's algorithm.

UNIT-III: Planar Graphs, Graph Coloring and Network Flows (15 hours)

Planar graphs, Euler's formula, Kuratowski theorem, Graph coloring, Applications of graph coloring, Circuit testing and facilities design, Flows and cuts, Max flow-min cut theorem, Matchings, Hall's theorem.

Essential Reading

1. Goodaire, Edgar G., & Parmenter, Michael M. (2011). Discrete Mathematics with Graph Theory (3rd ed.). Pearson Education Pvt. Ltd. Indian Reprint.

Suggestive Readings

- Bondy, J. A. & Murty, U.S.R. (2008), Graph Theory with Applications. Springer.
- Chartrand, Gary, & Zhang, P. (2012). A First Course in Graph Theory. Dover Publications.
- Diestel, R. (1997). Graph Theory (Graduate Texts in Mathematics). Springer Verlag.
- West, Douglas B. (2001). Introduction to graph theory (2nd ed.). Pearson India.

DISCIPLINE SPECIFIC ELECTIVE COURSE-2(iii): LINEAR PROGRAMMING

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Linear Programming	4	3	1	0	Class XII pass with Mathematics	NIL

Learning Objectives: The primary objective of this course is to introduce:

- The solution of linear programming problem using simplex method.
- The solution of transportation and assignment problems.
- Game theory which makes possible the analysis of the decision-making process of two interdependent subjects.

Learning Outcomes: This course will enable the students to:

- Learn about the simplex method used to find optimal solutions of linear optimization problems subject to certain constraints.
- Write the dual of a linear programming problem.

- Solve the transportation and assignment problems.
- Learn about solution of rectangular games using graphical method and dominance.
- Formulate game to a pair of associated prima-dual linear programming problems.

SYLLABUS OF DSE-2(iii)

UNIT-I: Linear Programming Problem, Simplex Method, and Duality (18 hours)

Standard form of the LPP, graphical method of solution, basic feasible solutions, and convexity; Introduction to the simplex method**:** Optimality criterion and unboundedness, Simplex tableau and examples, Artificial variables; Introduction to duality, Formulation of the dual problem with examples.

UNIT-II: Transportation and Assignment Problems (15 hours)

Definition of transportation problem, finding initial basic feasible solution using Northwest-corner method, Least-cost method, and Vogel approximation method; Algorithm for solving transportation problem; Hungarian method of solving assignment problem.

UNIT-III: Two-Person Zero-Sum Games (12 hours)

Introduction to game theory, rectangular games, Mixed strategies, Dominance principle; Formulation of game to primal and dual linear programming problems.

Essential Readings

1. Thie, Paul R., & Keough, G. E. (2014). An Introduction to Linear Programming and Game Theory. (3rd ed.). Wiley India Pvt. Ltd.
2. Taha, Hamdy A. (2017). Operations Research: An Introduction (10th ed.). Pearson.

Suggestive Readings

- Hadley, G. (1997). Linear Programming. Narosa Publishing House. New Delhi.
- Hillier, F. S., & Lieberman, G. J. (2021). Introduction to Operations Research (11th ed.). McGraw-Hill Education (India) Pvt. Ltd.

COMMON POOL OF GENERIC ELECTIVES (GE) Semester-IV COURSES OFFERED BY DEPARTMENT OF MATHEMATICS

Category-IV

GENERIC ELECTIVES (GE-4(i)): ELEMENTS OF REAL ANALYSIS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Elements of Real Analysis	4	3	1	0	Class XII pass with Mathematics	NIL

Learning Objectives: The primary objective of this course is to introduce:

- The real line with algebraic, order and completeness properties.
- Convergence and divergence of sequences and series of real numbers with applications.

Learning Outcomes: This course will enable the students to:

- Understand the basic properties of the set of real numbers, including completeness and Archimedean with some consequences.
- Recognize bounded, convergent, monotonic and Cauchy sequences
- Learn to apply various tests such as limit comparison, ratio, root, and alternating series tests for convergence and absolute convergence of infinite series of real numbers.

SYLLABUS OF GE-4(i)

UNIT-I: Basic Properties of the Set of Real Numbers (12 hours)

Field and order properties of $\mathbb{R}$, basic properties and inequalities of the absolute value of a real number, bounded above and bounded below sets, Suprema and infima, The completeness axiom and the Archimedean property of $\mathbb{R}$.

UNIT-II: Real Sequences (18 hours)

Convergence of a real sequence, Algebra of limits, The squeeze principle and applications, Monotone sequences, Monotone convergence theorem and applications, Cauchy sequences, Cauchy criterion for convergence and applications.

UNIT-III: Infinite Series of Real Numbers (15 hours)

Convergence and divergence of infinite series of real numbers, Necessary condition for convergence, Cauchy criterion for convergence of series, Tests for convergence of positive term series, Applications of the integral test, Comparison tests, D'Alembert's ratio test, Cauchy's nth root test, Raabe's test; Alternating series, Leibniz alternating series test, Absolute and conditional convergence.

Essential Reading

1. Denlinger, Charles G. (2011). Elements of Real Analysis. Jones & Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

Suggestive Readings

- Bartle, Robert G., & Sherbert, Donald R. (2011). Introduction to Real Analysis (4th ed.). John Wiley & Sons. Wiley India Edition 2015.
- Bilodeau, Gerald G., Thie, Paul R., & Keough, G. E. (2010). An Introduction to Analysis (2nd ed.). Jones & Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

GENERIC ELECTIVES (GE-4(ii)): LINEAR PROGRAMMING

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Linear Programming	4	3	1	0	Class XII pass with Mathematics	NIL

Learning Objectives: The primary objective of this course is to introduce:

- The solution of linear programming problem using simplex method.
- The solution of transportation and assignment problems.
- Game theory which makes possible the analysis of the decision-making process of two interdependent subjects.

Learning Outcomes: This course will enable the students to:

- Learn about the simplex method used to find optimal solutions of linear optimization problems subject to certain constraints.
- Write the dual of a linear programming problem.
- Solve the transportation and assignment problems.
- Learn about solution of rectangular games using graphical method and dominance.
- Formulate game to a pair of associated prima-dual linear programming problems.

SYLLABUS OF GE-4(ii)

UNIT-I: Linear Programming Problem, Simplex Method, and Duality (18 hours)

Standard form of the LPP, graphical method of solution, basic feasible solutions, and convexity; Introduction to the simplex method**:** Optimality criterion and unboundedness, Simplex tableau and examples, Artificial variables; Introduction to duality, Formulation of the dual problem with examples.

UNIT-II: Transportation and Assignment Problems (15 hours)
Definition of transportation problem, finding initial basic feasible solution using Northwest-corner method, Least-cost method, and Vogel approximation method; Algorithm for solving transportation problem; Hungarian method of solving assignment problem.

UNIT-III: Two-Person Zero-Sum Games (12 hours)
Introduction to game theory, rectangular games, Mixed strategies, Dominance principle; Formulation of game to primal and dual linear programming problems.

Essential Readings

1. Thie, Paul R., & Keough, G. E. (2014). An Introduction to Linear Programming and Game Theory. (3rd ed.). Wiley India Pvt. Ltd.
2. Taha, Hamdy A. (2017). Operations Research: An Introduction (10th ed.). Pearson.

Suggestive Readings

- Hadley, G. (1997). Linear Programming. Narosa Publishing House. New Delhi.
- Hillier, F. S., & Lieberman, G. J. (2021). Introduction to Operations Research (11th ed.). McGraw-Hill Education (India) Pvt. Ltd.

DEPARTMENT OF MATHEMATICS

Category-I

B.Sc. (Hons.) Mathematics Semester-V

DISCIPLINE SPECIFIC CORE COURSE – 13: METRIC SPACES

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Metric Spaces	4	3	1	0	Class XII pass with Mathematics	DSC-2: Real Analysis DSC-5: Calculus

Learning Objectives: The objective of the course is to introduce:

- The usual idea of distance into an abstract form on any set of objects, maintaining its inherent characteristics, and the resulting consequences.
- The two important topological properties, namely connectedness, and compactness of metric spaces with their characterizations.

Learning Outcomes: This course will enable the students to:

- Learn various natural and abstract formulations of distance on the sets of usual or unusual entities. Become aware one such formulations leading to metric spaces.
- Analyse how a theory advances from a particular frame to a general frame.
- Appreciate the mathematical understanding of various geometrical concepts, viz. balls or connected sets etc. in an abstract setting.
- Know about Banach fixed point theorem, whose far-reaching consequences have resulted into an independent branch of study in analysis, known as fixed point theory.

SYLLABUS OF DSC-13

UNIT – I: Topology of Metric Spaces (18 hours)

Definition, examples, sequences and Cauchy sequences, Complete metric space; Open and closed balls, Neighborhood, Open set, Interior of a set, Limit point of a set, Derived set, Closed set, Closure of a set, Diameter of a set, Cantor's theorem, Subspaces.

UNIT – II: Continuity and Uniform Continuity in Metric Spaces (15 hours)

Continuous mappings, Sequential criterion and other characterizations of continuity, Uniform continuity; Homeomorphism, Isometry and equivalent metrics, Contraction mapping, Banach fixed point theorem.

UNIT – III: Connectedness and Compactness (12 hours)

Connectedness, Connected subsets of $\mathbb{R}$, Connectedness and continuous mappings, Compactness and boundedness, Characterizations of compactness, Continuous functions on compact spaces.

Essential Reading

3. Shirali, Satish & Vasudeva, H. L. (2009). Metric Spaces. Springer. Indian Reprint 2019.

Suggestive Readings

- Kumaresan, S. (2014). Topology of Metric Spaces (2nd ed.). Narosa Publishing House. New Delhi.
- Rudin, Walter. Principles of mathematical Analysis (3rd ed.).
- Simmons, George F. (2004). Introduction to Topology and Modern Analysis. McGraw-Hill Education. New Delhi.

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

DISCIPLINE SPECIFIC CORE COURSE – 14: RING THEORY

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Ring Theory	4	3	1	0	Class XII pass with Mathematics	DSC-7: Group Theory

Learning Objectives: The primary objective of this course is to:

- Introduce the fundamental theory of rings, and their homomorphisms.
- Develop the basic concepts of polynomial rings and irreducibility tests for polynomials over the ring of integers, and rational numbers.
- Introduce polynomial analog of a prime number.
- Describe polynomial rings, principal ideal domains, Euclidean domains and unique factorization domains, and their relationships.

Learning Outcomes: This course will enable the students to:

- Learn about the fundamental concept of rings, integral domains, and fields.
- Know about ring homomorphisms and isomorphisms theorems of rings, and construct quotient fields for integral domains.
- Appreciate the significance of unique factorization in rings and integral domains.
- Apply several criteria for determining when polynomials with integer coefficients have rational roots or are irreducible over the field of rational numbers.

SYLLABUS OF DSC-14

UNIT – I: Introduction to Rings and Ideals (18 hours)

Definition and examples of rings, Properties of rings, Subrings, Integral domains and fields, Characteristic of a ring; Ideals, operations on ideals, ideal generated by a set and properties, Factor rings, Prime ideals and maximal ideals, Principal ideal domains.

UNIT – II: Ring Homomorphisms and Polynomial Rings (15 hours)

Definition, examples and properties of ring homomorphisms; First, second and third isomorphism theorems for rings; The field of quotients; Polynomial rings over commutative rings, Division algorithm and consequences.

UNIT–III: Unique Factorization Domain and Divisibility in Integral Domains (12 hours)

Factorization of polynomials, Reducibility tests, Mod p Irreducibility test, Eisenstein's criterion, Unique factorization in $\mathbb{Z}[x]$; Divisibility in integral domains, Irreducibles, Primes, Unique factorization domains, Euclidean domains.

Essential Readings

1. Gallian, Joseph. A. (2017). Contemporary Abstract Algebra (9th ed.). Cengage Learning India Private Limited, Delhi. Indian Reprint 2021.
2. Dummit, David S. & Foote, Richard M. (2016). Abstract Algebra (3rd ed.). Student Edition. Wiley India.

Suggestive Readings

- Herstein, I. N. (2006). Topics in Algebra (2nd ed.). Wiley Student Edition. India.
- Hungerford, Thomas W. (2012). Abstract Algebra: An Introduction (3rd ed.). Cengage Learning.

DISCIPLINE SPECIFIC CORE COURSE – 15: PARTIAL DIFFERENTIAL EQUATIONS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Partial Differential Equations	4	3	0	1	Class XII pass with Mathematics	DSC-6: Ordinary Differential Equations

Learning Objectives: The main objective of this course is to introduce:

- Basic concepts of first and second order linear/nonlinear partial differential equations.
- Modeling of wave equation, heat equation, Burgers equation, traffic flow and their solutions.

Learning Outcomes: The course will enable the students to learn:

- The method of characteristics and reduction to canonical forms to solve first and second order linear/nonlinear partial differential equations.
- The macroscopic modeling of the traffic flow, where the focus will be on modeling the density of cars and their flow, rather than modeling individual cars and their velocity.
- The Cauchy problem and solutions of wave equations with initial boundary-value problems, and non-homogeneous boundary conditions.

SYLLABUS OF DSC-15

UNIT – I: First Order Partial Differential Equations (15 hours)

Basic concepts, classification, construction, and geometrical interpretation; Method of characteristics and general solutions, Cauchy problem for a first-order PDE, Canonical forms of first-order linear equations; Method of separation of variables; Charpit's method for solving non-linear PDEs.

UNIT – II: Classification and Solutions of Second-Order Linear PDEs (12 hours)
Classification (hyperbolic, parabolic, and elliptic), reduction to canonical forms, and general solutions of second-order linear PDEs; Higher order linear partial differential equations with constant coefficients.

UNIT – III: Applications of Partial Differential Equations (18 hours)
Mathematical models: The vibrating string, vibrating membrane, conduction of heat in solids, the gravitational potential, conservation laws and the Burgers equation, Traffic flow; Cauchy problem and wave equations: Solutions of homogeneous wave equations with initial boundary-value problems, and non-homogeneous boundary conditions, Cauchy problem for non-homogeneous wave equations.

Essential Readings

1 Myint-U, Tyn & Debnath, Lokenath. (2007). Linear Partial Differential Equations for Scientists and Engineers (4th ed.). Birkhäuser. Indian Reprint.
2 Sneddon, Ian N. (2006). Elements of Partial Differential Equations, Dover Publications. Indian Reprint.

Suggestive Readings

- Abell, Martha & Braselton, J.P. (2004) Differential Equations with Mathematica, Elsevier, Academic Press, Third Edition.
- Stavroulakis, Ioannis P & Tersian, Stepan A. (2004). Partial Differential Equations: An Introduction with Mathematica and MAPLE (2nd ed.). World Scientific.

Practical (30 hours)- Practical / Lab work to be performed in a Computer Lab:
Modeling of the following similar problems using SageMath/Python/Mathematica/ MATLAB/Maple/Maxima/Scilab:

1. General solution of first and second order partial differential equations.
2. Solution and plotting of Cauchy problem for first order PDEs.
3. Plotting the characteristics for the first order partial differential equations.
4. Solution of vibrating string problem using D'Alembert formula with initial conditions.
5. Solution of heat equation $u_t = k\, u_{xx}$ with initial conditions.
6. Solution of one-dimensional wave equation with initial conditions:
   i. $u(x,0) = f(x),\ u_t(x,0) = g(x),\ x \in \mathbb{R}, t > 0$
   ii. $u(x,0) = f(x),\ u_t(x,0) = g(x),\ u(0,t) = 0,\ x \in \mathbb{R}, t > 0$
   iii. $u(x,0) = f(x),\ u_t(x,0) = g(x),\ u_x(0,t) = 0,\ x \in \mathbb{R}, t > 0$
7. Solution of traffic flow problem with given initial conditions, and plotting of the characteristic base curves and the traffic density function.

B.Sc. (Hons) Mathematics, Semester-V, DSE-Courses

DISCIPLINE SPECIFIC ELECTIVE COURSE – 3(i): MATHEMATICAL DATA SCIENCE

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title &	Credits	Credit distribution of the course	Eligibility	Pre-requisite of

Code		Lecture	Tutorial	Practical/ Practice	criteria	the course (if any)
Mathematical Data Science	4	3	0	1	Class XII pass with Mathematics	Basic knowledge of R/Python DSC-3: Probability & Statistics

Learning Objectives: The main objective of this course is to:

- Introduce various types of data and their sources, along with steps involved in data science case-study, including problems with data and their rectification and creation methods.
- Cover dimensionality reduction techniques, clustering algorithms and classification methods.

Learning Outcomes: The course will enable the students to:

- Gain a comprehensive understanding of data science, its mathematical foundations including practical applications of regression, principal component analysis, singular value decomposition, clustering, support vector machines, and *k*-NN classifiers.
- Demonstrate data analysis and exploration, linear regression techniques such as simple, multiple explanatory variables, cross-validation and regularization using R/Python.
- Use real-world datasets to practice dimensionality reduction techniques such as PCA, SVD, and multidimensional scaling using R/Python.

SYLLABUS OF DSE-3(i)

UNIT-I: Principles of Data Science (12 hours)

Types of Data: nominal, ordinal, interval, and ratio; Steps involved in data science case-study: question, procurement, exploration, modeling, and presentation; Structured and unstructured data: streams, frames, series, survey results, scale and source of data – fixed, variable, high velocity, exact and implied/inferred; Overview of problems with data – dirty and missing data in tabular formats – CSV, data frames in R/Pandas, anomaly detection, assessing data quality, rectification and creation methods, data hygiene, meta-data for inline data-description-markups such as XML and JSON; Overview of other data-source formats – SQL, pdf, Yaml, HDF5, and Vaex.

Unit-II: Mathematical Foundations (15 hours)

Model driven data in R^n, Log-likelihoods and MLE, Chebyshev, and Chernoff-Hoeffding inequalities with examples, Importance sampling; Norms in Vector Spaces– Euclidean, and metric choices; Types of distances: Manhattan, Hamming, Mahalanobis, Cosine and angular distances, KL divergence; Distances applied to sets– Jaccard, and edit distances; Modeling text with distances; Linear Regression: Simple, multiple explanatory variables, polynomial, cross-validation, regularized, Lasso, and matching pursuit; Gradient descent.

Unit-III: Dimensionality Reduction, Clustering and Classification (18 hours)

Problem of dimensionality, Principal component analysis, Singular value decomposition (SVD), Best *k*-rank approximation of a matrix, Eigenvector and eigenvalues relation to SVD, Multidimensional scaling, Linear discriminant analysis; Clustering: Voronoi diagrams, Delaunay triangulation, Gonzalez’s algorithm for *k*-center clustering, Lloyd’s algorithm for *k*-means clustering, Mixture of Gaussians, Hierarchical clustering, Density-based clustering and outliers, Mean shift clustering; Classification: Linear classifiers, Perceptron algorithm, Kernels, Support vector machines, and *k*-nearest neighbors (*k*-NN) classifiers.

Essential Readings

1. Mertz, David. (2021). Cleaning Data for Effective Data Science, Packt Publishing.
2. Ozdemir, Sinan. (2016). Principles of Data Science, Packt Publishing.
3. Phillips, Jeff M. (2021). Mathematical Foundations for Data Analysis, Springer. (https://mathfordata.github.io/).

Suggestive Readings

- Frank Emmert-Streib, et al. (2022). Mathematical Foundations of Data Science Using R. (2nd ed.). De Gruyter Oldenbourg.
- Wes McKinney. (2022). Python for Data Analysis (3rd ed.). O'Reilly.
- Wickham, Hadley, et al. (2023). R for Data Science (2nd ed.). O'Reilly.

Practical (30 hours)- Practical work to be performed in Computer Lab using R/Python:

1. To explore different types data (nominal, ordinal, interval, ratio) and identify their properties.
2. To deal with dirty and missing data, such as imputation, deletion, and data normalization.
3. Use the real-world datasets (https://data.gov.in/) to demonstrate the following:
   a) Data analysis and exploration, linear regression techniques such as simple, multiple explanatory variables, cross-validation, and regularization.
   b) Dimensionality reduction techniques such as principal component analysis, singular value decomposition (SVD), and multidimensional scaling.
   c) Clustering algorithms such as *k*-means, hierarchical, and density-based clustering and evaluate the quality of the clustering results.
   d) Classification methods such as linear classifiers, support vector machines (SVM), and *k*-nearest neighbors (*k*-NN).

DISCIPLINE SPECIFIC ELECTIVE COURSE – 3(ii): LINEAR PROGRAMMING AND APPLICATIONS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Linear Programming and Applications	4	3	1	0	Class XII pass with Mathematics	DSC-4: Linear Algebra

Learning Objectives: Primary objective of this course is to introduce:

- Simplex Method for linear programming problems.
- Dual linear programming problems.
- The applications of linear Programming to transportation, assignment, and game theory.

Learning Outcomes: The course will enable the students to:

- Learn about the basic feasible solutions of linear programming problems.
- Understand the theory of the simplex method to solve linear programming problems.
- Learn about the relationships between the primal and dual problems.
- Solve transportation and assignment problems.
- Understand two-person zero sum game, games with mixed strategies and formulation of game to primal and dual linear programing problems to solve using duality.

SYLLABUS OF DSE-3(ii)

UNIT– I: Introduction to Linear Programming (12 hours)

Linear programming problem: Standard, Canonical and matrix forms, Geometric solution; Convex and polyhedral sets, Hyperplanes, Extreme points; Basic solutions, Basic feasible solutions, Correspondence between basic feasible solutions and extreme points.

UNIT– II: Optimality and Duality Theory of Linear Programming Problem (18 hours)

Simplex method: Optimal solution, Termination criteria for optimal solution of the linear programming problem, Unique and alternate optimal solutions, Unboundedness; Simplex algorithm and its tableau format; Artificial variables, Two-phase method, Big-M method. Duality Theory: Motivation and formulation of dual problem, Primal-Dual relationships, Fundamental theorem of duality; Complementary slackness.

UNIT – III: Applications (15 hours)

Transportation Problem: Definition and formulation, Northwest-corner, Least- cost, and Vogel’s approximation methods of finding initial basic feasible solutions; Algorithm for solving transportation problem.

Assignment Problem: Mathematical formulation and Hungarian method of solving.

Game Theory: Two-person zero sum game, Games with mixed strategies, Formulation of game to primal and dual linear programming problems, Solution of games using duality.

Essential Readings

1. Bazaraa, Mokhtar S., Jarvis, John J., & Sherali, Hanif D. (2010). Linear Programming and Network Flows (4th ed.). John Wiley and Sons. Indian Reprint.
2. Hillier, Frederick S. & Lieberman, Gerald J. (2021). Introduction to Operations Research (11th ed.). McGraw-Hill Education (India) Pvt. Ltd.
3. Taha, Hamdy A. (2017). Operations Research: An Introduction (10th ed.). Pearson.

Suggestive Readings

- Hadley, G. (1997). Linear Programming. Narosa Publishing House. New Delhi.
- Thie, Paul R., & Keough, G. E. (2008). An Introduction to Linear Programming and Game Theory. (3rd ed.). Wiley India Pvt. Ltd. Indian Reprint 2014.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 3(iii): MATHEMATICAL STATISTICS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/		

				Practice		
Mathematical Statistics	**4**	**3**	**1**	**0**	**Class XII pass with Mathematics**	**DSC-3: Probability & Statistics DSC-11: Multivariate Calculus**

Learning Objectives: The main objective of this course is to introduce:

- The joint behavior of several random variables theoretically and through illustrative practical examples.
- The theory underlying modern statistics to give the student a solid grounding in (mathematical) statistics and the principles of statistical inference.
- The application of the theory to the statistical modeling of data from real applications, including model identification, estimation, and interpretation.
- The idea of Fisher information to find the minimum possible variance for an unbiased estimator, and to show that the MLE is asymptotically unbiased and normal.

Learning Outcomes: The course will enable the students to:

- Understand joint distributions of random variables including the bivariate normal distribution.
- Estimate model parameters from the statistical inference based on point estimation and hypothesis testing.
- Apply Rao-Blackwell theorem for improving an estimator, and Cramér-Rao inequality to find lower bound on the variance of unbiased estimators of a parameter.
- Understand the theory of linear regression models and contingency tables.

SYLLABUS OF DSE - 3(iii)

UNIT–I: Joint Probability Distributions (15 hours)

Joint probability mass function for two discrete random variables, Marginal probability mass function, Joint probability density function for two continuous random variables, Marginal probability density function, Independent random variables; Expected values, covariance, and correlation; Linear combination of random variables and their moment generating functions; Conditional distributions and conditional expectation, Laws of total expectation and variance; Bivariate normal distribution.

UNIT-II: Sampling Distributions and Point Estimation (15 hours)

Distribution of important statistics such as the sample totals, sample means, and sample proportions, Central limit theorem, Law of large numbers; Chi-squared, t, and F distributions; Distributions based on normal random samples; Concepts and criteria for point estimation, The methods of moments and maximum likelihood estimation (MLE); Assessing estimators: Accuracy and precision, Unbiased estimation, Consistency and sufficiency, The Neyman factorization theorem, Rao-Blackwell theorem, Fisher Information, The Cramér-Rao inequality, Efficiency,

UNIT-III: Confidence Intervals, Tests of Hypotheses and Linear Regression Analysis (15 hours)

Interval estimation and basic properties of confidence intervals, One-sample *t* confidence interval, Confidence intervals for a population proportion and population variance.
Statistical hypotheses and test procedures, One-sample tests about a population mean and a population proportion, *P*-values for tests; The simple linear regression model and its estimating parameters; Chi-squared goodness-of-fit tests, Two-way contingency tables.

Essential Reading

1. Devore, Jay L., Berk, Kenneth N. & Carlton Matthew A. (2021). Modern Mathematical Statistics with Applications. (3rd ed.). Springer.

Suggestive Readings

- Devore, Jay L. (2016). Probability and Statistics for Engineering and the Sciences. Ninth edition, Cengage Learning India Private Limited, Delhi. Fourth impression 2022.
- Hogg, Robert V., McKean, Joseph W., & Craig, Allen T. (2019). Introduction to Mathematical Statistics. Eighth edition, Pearson. Indian Reprint 2020.
- Mood, A.M., Graybill, F.A., & Boes, D.C. (1974). Introduction the Theory of Statistics (3rd ed.). Tata McGraw Hill Pub. Co. Ltd. Reprinted 2017.
- Wackerly, Dennis D., Mendenhall III, William & Scheaffer, Richard L. (2008). Mathematical Statistics with Applications. 7th edition, Cengage Learning.

B.A. (Prog.) Semester-V with Mathematics as Major

Category-II

DISCIPLINE SPECIFIC CORE COURSE (DSC-5): LINEAR PROGRAMMING

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Linear Programming	4	3	1	0	Class XII pass with Mathematics	NIL

Learning Objectives: The primary objective of this course is to introduce:

- The solution of linear programming problem using simplex method.
- The solution of transportation and assignment problems.
- Game theory which makes possible the analysis of the decision-making process of two interdependent subjects.

Learning Outcomes: This course will enable the students to:

- Learn about the simplex method used to find optimal solutions of linear optimization problems subject to certain constraints.
- Write the dual of a linear programming problem.
- Solve the transportation and assignment problems.
- Learn about solution of rectangular games using graphical method and dominance.
- Formulate game to a pair of associated prima-dual linear programming problems.

SYLLABUS OF DSC-5

UNIT-I: Linear Programming Problem, Simplex Method, and Duality (18 hours)

Standard form of the LPP, graphical method of solution, basic feasible solutions, and convexity; Introduction to the simplex method: Optimality criterion and unboundedness, Simplex tableau and examples, Artificial variables; Introduction to duality, Formulation of the dual problem with examples.

UNIT-II: Transportation and Assignment Problems (15 hours)

Definition of transportation problem, finding initial basic feasible solution using Northwest-corner method, Least-cost method, and Vogel approximation method; Algorithm for solving transportation problem; Hungarian method of solving assignment problem.

UNIT-III: Two-Person Zero-Sum Games (12 hours)

Introduction to game theory, rectangular games, Mixed strategies, Dominance principle; Formulation of game to primal and dual linear programming problems.

Essential Readings

3. Thie, Paul R., & Keough, G. E. (2014). An Introduction to Linear Programming and Game Theory. (3rd ed.). Wiley India Pvt. Ltd.
4. Taha, Hamdy A. (2017). Operations Research: An Introduction (10th ed.). Pearson.

Suggestive Readings

- Hadley, G. (1997). Linear Programming. Narosa Publishing House. New Delhi.
- Hillier, F. S., & Lieberman, G. J. (2021). Introduction to Operations Research (11th ed.).McGraw-Hill Education (India) Pvt. Ltd.

DISCIPLINE SPECIFIC CORE COURSE – 5 (Discipline A-5): ELEMENTS OF REAL ANALYSIS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Elements of Real Analysis	4	3	1	0	Class XII pass with Mathematics	NIL

Learning Objectives: The primary objective of this course is to introduce:
- The real line with algebraic, order and completeness properties.
- Convergence and divergence of sequences and series of real numbers with applications.

Learning Outcomes: This course will enable the students to:
- Understand the basic properties of the set of real numbers, including completeness and Archimedean with some consequences.
- Recognize bounded, convergent, monotonic and Cauchy sequences
- Learn to apply various tests such as limit comparison, ratio, root, and alternating series tests for convergence and absolute convergence of infinite series of real numbers.

SYLLABUS OF DISCIPLINE A-5

UNIT-I: Basic Properties of the Set of Real Numbers (12 hours)

Field and order properties of $\mathbb{R}$, basic properties and inequalities of the absolute value of a real number, bounded above and bounded below sets, Suprema and infima, The completeness axiom and the Archimedean property of $\mathbb{R}$.

UNIT-II: Real Sequences (18 hours)

Convergence of a real sequence, Algebra of limits, The squeeze principle and applications, Monotone sequences, Monotone convergence theorem and applications, Cauchy sequences, Cauchy criterion for convergence and applications.

UNIT-III: Infinite Series of Real Numbers (15 hours)

Convergence and divergence of infinite series of real numbers, Necessary condition for convergence, Cauchy criterion for convergence of series, Tests for convergence of positive term series, Applications of the integral test, Comparison tests, D'Alembert's ratio test, Cauchy's *n*th root test, Raabe's test; Alternating series, Leibniz alternating series test, Absolute and conditional convergence.

Essential Reading

1. Denlinger, Charles G. (2011). Elements of Real Analysis. Jones & Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

Suggestive Readings

- Bartle, Robert G., & Sherbert, Donald R. (2011). Introduction to Real Analysis (4th ed.). John Wiley & Sons. Wiley India Edition 2015.
- Bilodeau, Gerald G., Thie, Paul R., & Keough, G. E. (2010). An Introduction to Analysis (2nd ed.). Jones & Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

DSE Courses of B.A. (Prog.) Semester-V
Category-II

DISCIPLINE SPECIFIC ELECTIVE COURSE – 1(i): COMBINATORICS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Combinatorics	4	3	1	0	Class XII pass with Mathematics	NIL

Learning Objectives: The primary objective of this course is to:
- Introduce various techniques of permutations, combinations, and inclusion-exclusion.
- Learn basic models of generating functions and recurrence relations in their application to the theory of integer partitions.

Learning Outcomes: After completing the course, student will:
- Enhance the mathematical logical skills by learning different enumeration techniques.
- Be able to apply these techniques in solving problems in other areas of mathematics.
- Be trained to provide reasoning and arguments to justify conclusions.

SYLLABUS OF DSE-1(i)

UNIT – I: Basics of Combinatorics (15 hours)
Basic counting principles, Permutations and Combinations (with and without repetitions), Binomial coefficients, Multinomial coefficients, Counting subsets of size *k*; Set-partitions, The inclusion-exclusion principle and applications.

UNIT – II: Generating Functions and Recurrence Relations (18 hours)
Generating functions: Generating function models, Calculating coefficients of generating functions, Polynomial expansions, Binomial identity, Exponential generating functions. Recurrence relations: Recurrence relation models, Divide-and-conquer relations, Solution of linear recurrence relations, Solutions by generating functions.

UNIT – III: Partition (12 hours)
Partition theory of integers: Ordered partition, Unordered partition, Ferrers diagram, Conjugate of partition, Self-conjugate partition, Durfee square, Euler's pentagonal theorem.

Essential Readings

1. Sane, Sharad S. (2013). Combinatorial Techniques. Hindustan Book Agency (India).
2. Tucker, Alan (2012). Applied Combinatorics (6th ed.). John Wiley & Sons, Inc.

Suggestive Readings

- Brualdi, Richard A. (2009). Introductory Combinatorics (5th ed.). Pearson Education.
- Cameron, Peter J. (1994). Combinatorics: Topics, Techniques, Algorithms. Cambridge University Press.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 1(ii): ELEMENTS OF NUMBER THEORY

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Elements of Number Theory	4	3	1	0	Class XII pass with Mathematics	NIL

Learning Objectives: The primary objective of this course is to introduce:

- The Euclidean algorithm and linear Diophantine equations, the Fundamental theorem of arithmetic and some of the open problems of number theory viz. the Goldbach conjecture.
- The modular arithmetic, linear congruence equations, system of linear congruence equations, arithmetic functions, and multiplicative functions, e.g., Euler's Phi-function.
- Introduction of the simple encryption and decryption techniques, and the numbers of specific forms viz. Mersenne numbers, Fermat numbers etc.

Learning Outcomes: This course will enable the students to:

- Get familiar with the basic number-theoretic techniques.
- Comprehend some of the open problems in number theory.
- Learn the properties and use of number-theoretic functions and special types of numbers.
- Acquire knowledge about public-key cryptosystems, particularly RSA.

SYLLABUS OF DSE-1(ii)

UNIT – I: Divisibility and Prime Numbers (12 hours)

Revisiting: The division algorithm, divisibility and the greatest common divisor. Euclid's lemma; The Euclidean algorithm, Linear Diophantine equations; The Fundamental theorem of Arithmetic, The sieve of Eratosthenes, Euclid theorem and the Goldbach conjecture; The Fibonacci sequence and its nature.

UNIT – II: Theory of Congruences and Number-Theoretic Functions (21 hours)

Congruence relation and its basic properties, Linear congruences and the Chinese remainder theorem, System of linear congruences in two variables; Fermat's little theorem and its generalization, Wilson's theorem and its converse; Number-theoretic functions for sum and the number of divisors of a positive integer, Multiplicative functions, The greatest integer function; Euler's Phi-function and its properties.

UNIT – III: Public Key Encryption and Numbers of Special Form (12 hours)

Basics of cryptography, Hill's cipher, Public-key cryptosystems and RSA encryption and decryption technique; Introduction to perfect numbers, Mersenne numbers and Fermat numbers.

Essential Reading

1. Burton, David M. (2011). Elementary Number Theory (7th ed.). McGraw-Hill Education Pvt. Ltd. Indian Reprint 2017.

Suggestive Readings

- Jones, G. A., & Jones, J. Mary. (2005). Elementary Number Theory. Springer Undergraduate Mathematics Series (SUMS). Indian Reprint.
- Robbins, Neville (2007). Beginning Number Theory (2nd ed.). Narosa Publishing House Pvt. Ltd. Delhi.
- Rosen, Kenneth H. (2011). Elementary Number Theory and its Applications (6th ed.). Pearson Education. Indian Reprint 2015.

Note: Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 1(iii): MATHEMATICAL PYTHON

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Mathematical Python	4	3	0	1	Class XII pass with Mathematics	Basic knowledge of Python

Learning Objectives: The Learning Objectives of this course are as follows:

- To be able to model and solve mathematical problems using Python Programs.
- To experience utility of open-source resources for numerical and symbolic mathematical software systems.

Learning Outcomes: This course will enable the students to use Python:

- For numerical and symbolic computation in mathematical problems from calculus, algebra, and geometry.
- To tabulate and plot diverse graphs of functions and understand tracing of shapes, geometries, and fractals.
- To prepare smart documents with LaTeX interface.

SYLLABUS OF DSE - 1(iii)

Theory

UNIT – I: Drawing Shapes, Graphing and Visualization (15 hours)

Drawing diverse shapes using code and Turtle; Using matplotlib and NumPy for data organization, Structuring and plotting lines, bars, markers, contours and fields, managing subplots and axes; Pyplot and subplots, Animations of decay, Bayes update, Random walk.

UNIT – II: Numerical and Symbolic Solutions of Mathematical Problems (18 hours)

NumPy for scalars and linear algebra on *n*-dimensional arrays; Computing eigenspace, Solving dynamical systems on coupled ordinary differential equations, Functional programming fundamentals using NumPy; Symbolic computation and SymPy: Differentiation and integration of functions, Limits, Solution of ordinary differential equations, Computation of eigenvalues, Solution of expressions at multiple points (lambdify), Simplification of expressions, Factorization, Collecting and canceling terms, Partial fraction decomposition, Trigonometric simplification, Exponential and logarithms, Series expansion and finite differences, Solvers, Recursive equations.

UNIT – III: Document Generation with Python and LaTeX (12 hours)

Pretty printing using SymPy; Pandas API for IO tools: interfacing Python with text/csv, HTML, LaTeX, XML, MSExcel, OpenDocument, and other such formats; Pylatex and writing document files from Python with auto-computed values, Plots and visualizations.

Practical (30 hours): Software labs using IDE such as Spyder and Python Libraries.

- Installation, update, and maintenance of code, troubleshooting.
- Implementation of all methods learned in theory.
- Explore and explain API level integration and working of two problems with standard Python code.

Essential Readings

1. Farrell, Peter (2019). Math Adventures with Python. No Starch Press. ISBN Number: 978-1-59327-867-0.
2. Farrell, Peter and et al. (2020). The Statistics and Calculus with Python Workshop. Packet Publishing Ltd. ISBN: 978-1-80020-976-3.
3. Saha, Amit (2015). Doing Math with Python. No Starch Press. ISBN: 978-1-59327-640-9

Suggestive Readings

- Morley, Sam (2022). Applying Math with Python (2nd ed.). Packet Publishing Ltd. ISBN: 978-1-80461-837-0
- Online resources and documentation on the libraries, such as:
    - https://matplotlib.org
    - https://sympy.org
    - https://pandas.pydata.org
    - https://numpy.org
    - https://pypi.org
    - https://patrickwalls.github.io/mathematicalpython/

Note: Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

B.Sc. (Prog.)/ BA (Prog.) Semester-V with Mathematics as non-Major

Category-III

DISCIPLINE SPECIFIC CORE COURSE – 5 (Discipline A-5): ELEMENTS OF REAL ANALYSIS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Elements of Real Analysis	4	3	1	0	Class XII pass with Mathematics	NIL

Learning Objectives: The primary objective of this course is to introduce:

- The real line with algebraic, order and completeness properties.
- Convergence and divergence of sequences and series of real numbers with applications.

Learning Outcomes: This course will enable the students to:

- Understand the basic properties of the set of real numbers, including completeness and Archimedean with some consequences.
- Recognize bounded, convergent, monotonic and Cauchy sequences
- Learn to apply various tests such as limit comparison, ratio, root, and alternating series tests for convergence and absolute convergence of infinite series of real numbers.

SYLLABUS OF DISCIPLINE A-5

UNIT-I: Basic Properties of the Set of Real Numbers (12 hours)

Field and order properties of $\mathbb{R}$, basic properties and inequalities of the absolute value of a real number, bounded above and bounded below sets, Suprema and infima, The completeness axiom and the Archimedean property of $\mathbb{R}$.

UNIT-II: Real Sequences (18 hours)

Convergence of a real sequence, Algebra of limits, The squeeze principle and applications, Monotone sequences, Monotone convergence theorem and applications, Cauchy sequences, Cauchy criterion for convergence and applications.

UNIT-III: Infinite Series of Real Numbers (15 hours)

Convergence and divergence of infinite series of real numbers, Necessary condition for convergence, Cauchy criterion for convergence of series, Tests for convergence of positive term series, Applications of the integral test, Comparison tests, D'Alembert's ratio test, Cauchy's *n*th root test, Raabe's test; Alternating series, Leibniz alternating series test, Absolute and conditional convergence.

Essential Reading

1. Denlinger, Charles G. (2011). Elements of Real Analysis. Jones & Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

Suggestive Readings

- Bartle, Robert G., & Sherbert, Donald R. (2011). Introduction to Real Analysis (4th ed.). John Wiley & Sons. Wiley India Edition 2015.
- Bilodeau, Gerald G., Thie, Paul R., & Keough, G. E. (2010). An Introduction to Analysis (2nd ed.). Jones & Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

B.Sc. (Physical Sciences/Mathematical Sciences) Sem-V with Mathematics as one of the Core Discipline

Category-III

DISCIPLINE SPECIFIC CORE COURSE – 5 (Discipline A-5): ELEMENTS OF REAL ANALYSIS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Elements of Real Analysis	4	3	1	0	Class XII pass with Mathematics	NIL

Learning Objectives: The primary objective of this course is to introduce:

- The real line with algebraic, order and completeness properties.
- Convergence and divergence of sequences and series of real numbers with applications.

Learning Outcomes: This course will enable the students to:

- Understand the basic properties of the set of real numbers, including completeness and Archimedean with some consequences.
- Recognize bounded, convergent, monotonic and Cauchy sequences
- Learn to apply various tests such as limit comparison, ratio, root, and alternating series tests for convergence and absolute convergence of infinite series of real numbers.

SYLLABUS OF DISCIPLINE A-5

UNIT-I: Basic Properties of the Set of Real Numbers (12 hours)

Field and order properties of $\mathbb{R}$, basic properties and inequalities of the absolute value of a real number, bounded above and bounded below sets, Suprema and infima, The completeness axiom and the Archimedean property of $\mathbb{R}$.

UNIT-II: Real Sequences (18 hours)

Convergence of a real sequence, Algebra of limits, The squeeze principle and applications, Monotone sequences, Monotone convergence theorem and applications, Cauchy sequences, Cauchy criterion for convergence and applications.

UNIT-III: Infinite Series of Real Numbers (15 hours)

Convergence and divergence of infinite series of real numbers, Necessary condition for convergence, Cauchy criterion for convergence of series, Tests for convergence of positive term series, Applications of the integral test, Comparison tests, D'Alembert's ratio test, Cauchy's nth root test, Raabe's test; Alternating series, Leibniz alternating series test, Absolute and conditional convergence.

Essential Reading

1. Denlinger, Charles G. (2011). Elements of Real Analysis. Jones & Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

Suggestive Readings

- Bartle, Robert G., & Sherbert, Donald R. (2011). Introduction to Real Analysis (4th ed.). John Wiley & Sons. Wiley India Edition 2015.
- Bilodeau, Gerald G., Thie, Paul R., & Keough, G. E. (2010). An Introduction to Analysis (2nd ed.). Jones & Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

DSE Courses of B.Sc. (Physical Sciences/Mathematical Sciences) Semester-V

Category-III

DISCIPLINE SPECIFIC ELECTIVE COURSE – 3(i): BIOMATHEMATICS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Biomathematics	4	3	1	0	Class XII pass with Mathematics	Discipline A-3: Differential Equations

Learning Objectives: The main objective of this course is to:

- Develop and analyse the models of the biological phenomenon with emphasis on population growth and predator-prey models.
- Interpret first-order autonomous systems of nonlinear differential equations using the Poincaré phase plane.
- Apply the basic concepts of probability to understand molecular evolution and genetics.

Learning Outcomes: The course will enable the students to:

- Get a better comprehension of mathematical models, utilised in biology.
- To identify and explain the findings from models of population studies, species' communication, adaptation, and dynamics of disorder.
- Create a basic model of molecular evolution by making use of probability and matrices.

SYLLABUS OF DSE-3(i)

UNIT – I: Mathematical Modeling for Biological Processes (15 hours)

Formulation a model through data, A continuous population growth model, Long-term behavior and equilibrium states, The Verhulst model for discrete population growth, Administration of drugs, Differential equation of chemical process and predator-prey model (Function response: Types I, II and III).

UNIT – II: Epidemic Model: Formulation and Analysis (15 hours)

Introduction to infectious disease, The SIS, SIR and SEIR models of the spread of an epidemic, Analyzing equilibrium states, Phase plane analysis, Stability of equilibrium points, Classifying the equilibrium state; Local stability, Limit cycles, Poincaré-Bendixson theorem.

UNIT – III: Bifurcation, Chaos and Modeling Molecular Evolution (15 hours)

Bifurcation, Bifurcation of a limit cycle, Discrete bifurcation and period-doubling, Chaos, Stability of limit cycles, Introduction of the Poincaré plane; Modeling molecular evolution: Matrix models of base substitutions for DNA sequences, Jukes-Cantor and Kimura models, Phylogenetic distances.

Essential Readings

4. Robeva, Raina S., et al. (2008). An Invitation to Biomathematics. Academic press.
5. Jones, D. S., Plank, M. J., & Sleeman, B. D. (2009). Differential Equations and Mathematical Biology (2nd ed.). CRC Press, Taylor & Francis Group.
6. Allman, Elizabeth S., & Rhodes, John A. (2004). Mathematical Models in Biology: An Introduction. Cambridge University Press.

Suggestive Readings

- Linda J. S. Allen (2007). An Introduction to Mathematical Biology. Pearson Education.
- Murray, J. D. (2002). Mathematical Biology: An Introduction (3rd ed.). Springer.
- Shonkwiler, Ronald W., & Herod, James. (2009). Mathematical Biology: An Introduction with Maple and MATLAB (2nd ed.). Springer.

DISCIPLINE SPECIFIC ELECTIVE COURSE-3(ii): MATHEMATICAL PYTHON

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Mathematical Python	4	3	0	1	Class XII pass with Mathematics	Basic knowledge of Python

Learning Objectives: The Learning Objectives of this course are as follows:

- To be able to model and solve mathematical problems using Python Programs.
- To experience utility of open-source resources for numerical and symbolic mathematical software systems.

Learning Outcomes: This course will enable the students to use Python:

- For numerical and symbolic computation in mathematical problems from calculus, algebra, and geometry.
- To tabulate and plot diverse graphs of functions and understand tracing of shapes, geometries, and fractals.
- To prepare smart documents with LaTeX interface.

SYLLABUS OF DSE - 3(ii)

Theory

UNIT – I: Drawing Shapes, Graphing and Visualization (15 hours)

Drawing diverse shapes using code and Turtle; Using matplotlib and NumPy for data organization, Structuring and plotting lines, bars, markers, contours and fields, managing subplots and axes; Pyplot and subplots, Animations of decay, Bayes update, Random walk.

UNIT – II: Numerical and Symbolic Solutions of Mathematical Problems (18 hours)

NumPy for scalars and linear algebra on *n*-dimensional arrays; Computing eigenspace, Solving dynamical systems on coupled ordinary differential equations, Functional programming fundamentals using NumPy; Symbolic computation and SymPy: Differentiation and integration of functions, Limits, Solution of ordinary differential equations, Computation of eigenvalues, Solution of expressions at multiple points (lambdify), Simplification of expressions, Factorization, Collecting and canceling terms, Partial fraction decomposition, Trigonometric simplification, Exponential and logarithms, Series expansion and finite differences, Solvers, Recursive equations.

UNIT – III: Document Generation with Python and LaTeX (12 hours)

Pretty printing using SymPy; Pandas API for IO tools: interfacing Python with text/csv, HTML, LaTeX, XML, MSExcel, OpenDocument, and other such formats; Pylatex and writing document files from Python with auto-computed values, Plots and visualizations.

Practical (30 hours): Software labs using IDE such as Spyder and Python Libraries.

- Installation, update, and maintenance of code, troubleshooting.
- Implementation of all methods learned in theory.
- Explore and explain API level integration and working of two problems with standard Python code.

Essential Readings

1. Farrell, Peter (2019). Math Adventures with Python. No Starch Press. ISBN Number: 978-1-59327-867-0.
2. Farrell, Peter and et al. (2020). The Statistics and Calculus with Python Workshop. Packet Publishing Ltd. ISBN: 978-1-80020-976-3.
3. Saha, Amit (2015). Doing Math with Python. No Starch Press. ISBN: 978-1-59327-640-9

Suggestive Readings

- Morley, Sam (2022). Applying Math with Python (2nd ed.). Packet Publishing Ltd. ISBN: 978-1-80461-837-0
- Online resources and documentation on the libraries, such as:
    - https://matplotlib.org
    - https://sympy.org
    - https://pandas.pydata.org
    - https://numpy.org
    - https://pypi.org
    - https://patrickwalls.github.io/mathematicalpython/

Note: Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

DISCIPLINE SPECIFIC ELECTIVE COURSE-3(iii): MECHANICS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Mechanics	4	3	1	0	Class XII pass with Mathematics	Discipline A-1: Topics in Calculus Discipline A-3: Differential Equations

Learning Objectives: The main objective of this course is to:

- Starting Newtonian laws, learning various technical notions which explains various states of motion under given forces.
- Deals with the kinematics and kinetics of the rectilinear and planar motions of a particle including constrained oscillatory motions of particles, projectiles, and planetary orbits.
- Understand hydrostatic pressure and thrust on plane surfaces.

Learning Outcomes: This course will enable the students to:

- Understand necessary conditions for the equilibrium of particles acted upon by various forces and learn the principle of virtual work for a system of coplanar forces.
- Apply the concepts of center of gravity, laws of static and kinetic friction.
- Learn that a particle moving under a central force describes a plane curve and know the Kepler's laws of the planetary motions.
- Evaluate the hydrostatic pressure at any given depth in a heavy homogeneous liquid at rest under gravity.

SYLLABUS OF DSE-3(iii)

UNIT – I: Statics (15 hours)

Fundamental laws of Newtonian mechanics, Law of parallelogram of forces, Equilibrium of a particle, Lamy's theorem, Equilibrium of a system of particles, External and internal forces, Couples, Reduction of a plane force system, Work, Principle of virtual work, Potential energy and conservative field, Mass centers, Centers of gravity, Friction.

UNIT – II: Dynamics (18 hours)

Kinemetics of a particle, Motion of a particle, Motion of a system, Principle of linear momentum, Motion of mass center, Principle of angular momentum, Motion relative to mass center, Principle of energy, D'Alembert's principle; Moving frames of reference, Frames of reference with uniform translational velocity, Frames of reference with constant angular velocity; Applications in plane dynamics- Motion of a projectile, Harmonic oscillators, General motion under central forces, Planetary orbits.

UNIT – III: Hydrostatics (12 hours)

Shearing stress, Pressure, Perfect fluid, Pressure at a point in a fluid, Transmissibility of liquid pressure, Compression, Specific gravity, Pressure of heavy fluid- Pressure at all points in a horizontal plane, Surface of equal density; Thrust on plane surfaces.

Essential Readings

3. Synge, J. L., & Griffith, B. A. (2017). Principles of Mechanics (3rd ed.). McGraw-Hill Education. Indian Reprint.
4. Ramsey, A. S. (2017). Hydrostatics. Cambridge University Press. Indian Reprint.

Suggestive Readings

- Roberts, A. P. (2003). Statics and Dynamics with Background Mathematics. Cambridge University Press.
- Ramsey, A. S. (1985). Statics (2nd ed.). Cambridge University Press.

COMMON POOL OF GENERIC ELECTIVES (GE) Semester-V COURSES OFFERED BY DEPARTMENT OF MATHEMATICS

Category-IV

GENERIC ELECTIVES (GE-5(i)): NUMERICAL METHODS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course
		Lecture	Tutorial	Practical/ Practice		
Numerical Methods	4	3	0	1	Class XII pass with Mathematics	NIL

Learning Objectives: The core purpose of the course is to:

- Acquaint students with various topics in numerical solutions of nonlinear equations in one variable, interpolation and approximation, numerical differentiation and integration, direct methods for solving linear systems, numerical solution of ordinary differential equations using Computer Algebra System (CAS).

Learning Outcomes: The course will enable the students to:

- Find the consequences of finite precision and the inherent limits of numerical methods.
- Appropriate numerical methods to solve algebraic and transcendental equations.
- Solve first order initial value problems of ODE's numerically using Euler methods.

SYLLABUS OF GE-5(i)

UNIT-I: Errors and Roots of Transcendental and Polynomial Equations (12 hours)

Errors: Roundoff error, Local truncation error, Global truncation error; Order of a method, Convergence, and terminal conditions; Bisection method, Secant method, Regula−Falsi method, Newton−Raphson method.

UNIT-II: Algebraic Linear Systems and Interpolation (18 hours)

Gaussian elimination method (with row pivoting); Iterative methods: Jacobi method, Gauss−Seidel method; Interpolation: Lagrange form, Newton form, Finite difference operators.

UNIT-III: Numerical Differentiation, Integration and ODE (15 hours)

First and second order numerical derivatives; Trapezoidal rule, Simpson's rule for numerical integration; Ordinary differential equation: Euler's, and Runge-Kutta method.

Essential Readings

1. Chapra, Steven C. (2018). Applied Numerical Methods with MATLAB for Engineers and Scientists (4th ed.). McGraw-Hill Education.
2. Fausett, Laurene V. (2009). Applied Numerical Analysis Using MATLAB. Pearson. India.
3. Jain, M. K., Iyengar, S. R. K., & Jain R. K. (2012). Numerical Methods for Scientific and Engineering Computation (6th ed.). New Age International Publishers. Delhi.

Suggestive Reading

- Bradie, Brian (2006). A Friendly Introduction to Numerical Analysis. Pearson Education India. Dorling Kindersley (India) Pvt. Ltd. Third Impression, 2011.

Note: Non programmable scientific calculator may be allowed in the University examination.

Practical (30 hours): Practical/Lab work to be performed in Computer Lab: Use of computer algebra software (CAS), for example Python/SageMath/Mathematica/MATLAB/ Maple/ Maxima/Scilab etc., for developing the following numerical programs:

1. Bisection method
2. Secant method and Regula−Falsi method
3. Newton-Raphson method
4. Gauss−Jacobi method and Gauss−Seidel method
5. Lagrange interpolation and Newton interpolation
6. Trapezoidal rule and Simpson's rule
7. Euler's, and Runge-Kutta methods for solving first order initial-value problems of ordinary differential equations.

GENERIC ELECTIVES (GE-5(ii)): MATHEMATICAL PYTHON

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course
		Lecture	Tutorial	Practical/ Practice		
Mathematical Python	4	3	0	1	Class XII pass with Mathematics	Basic knowledge of python

Learning Objectives: The Learning Objectives of this course are as follows:

- To be able to model and solve mathematical problems using Python Programs.
- To experience utility of open-source resources for numerical and symbolic mathematical software systems.

Learning Outcomes: This course will enable the students to use Python:

- For numerical and symbolic computation in mathematical problems from calculus, algebra, and geometry.
- To tabulate and plot diverse graphs of functions and understand tracing of shapes, geometries, and fractals.
- To prepare smart documents with LaTeX interface.

SYLLABUS OF GE-5(ii)

Theory

UNIT – I: Drawing Shapes, Graphing and Visualization (15 hours)

Drawing diverse shapes using code and Turtle; Using matplotlib and NumPy for data organization, Structuring and plotting lines, bars, markers, contours and fields, managing subplots and axes; Pyplot and subplots, Animations of decay, Bayes update, Random walk.

UNIT – II: Numerical and Symbolic Solutions of Mathematical Problems (18 hours)

NumPy for scalars and linear algebra on *n*-dimensional arrays; Computing eigenspace, Solving dynamical systems on coupled ordinary differential equations, Functional programming fundamentals using NumPy; Symbolic computation and SymPy: Differentiation and integration of functions, Limits, Solution of ordinary differential equations, Computation of eigenvalues, Solution of expressions at multiple points (lambdify), Simplification of expressions, Factorization, Collecting and canceling terms, Partial fraction decomposition, Trigonometric simplification, Exponential and logarithms, Series expansion and finite differences, Solvers, Recursive equations.

UNIT – III: Document Generation with Python and LaTeX (12 hours)

Pretty printing using SymPy; Pandas API for IO tools: interfacing Python with text/csv, HTML, LaTeX, XML, MSExcel, OpenDocument, and other such formats; Pylatex and writing document files from Python with auto-computed values, Plots and visualizations.

Practical (30 hours): Software labs using IDE such as Spyder and Python Libraries.

- Installation, update, and maintenance of code, troubleshooting.
- Implementation of all methods learned in theory.
- Explore and explain API level integration and working of two problems with standard Python code.

Essential Readings

1. Farrell, Peter (2019). Math Adventures with Python. No Starch Press. ISBN Number: 978-1-59327-867-0.
2. Farrell, Peter and et al. (2020). The Statistics and Calculus with Python Workshop. Packet Publishing Ltd. ISBN: 978-1-80020-976-3.
3. Saha, Amit (2015). Doing Math with Python. No Starch Press. ISBN: 978-1-59327-640-9

Suggestive Readings

- Morley, Sam (2022). Applying Math with Python (2nd ed.). Packet Publishing Ltd. ISBN: 978-1-80461-837-0
- Online resources and documentation on the libraries, such as:
  - https://matplotlib.org
  - https://sympy.org
  - https://pandas.pydata.org
  - https://numpy.org
  - https://pypi.org
  - https://patrickwalls.github.io/mathematicalpython/

Note: Examination scheme and mode shall be as prescribed by the Examination Branch, University of Delhi, from time to time.

DEPARTMENT OF MATHEMATICS

Category-I

B.Sc. (Hons.) Mathematics, Semester-VI

DISCIPLINE SPECIFIC CORE COURSE – 16: ADVANCED GROUP THEORY

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Advanced Group Theory	4	3	1	0	Class XII pass with Mathematics	DSC-7: Group Theory

Learning Objectives: The objective of the course is to introduce:

- The concept of group actions.
- Sylow's Theorem and its applications to groups of various orders.
- Composition series and Jordan-Hölder theorem.

Learning Outcomes: This course will enable the students to:

- Understand the concept of group actions and their applications.
- Understand finite groups using Sylow's theorem.
- Use Sylow's theorem to determine whether a group is simple or not.
- Understand and determine if a group is solvable or not.

SYLLABUS OF DSC-16

UNIT – I: Group Actions (18 hours)

Definition and examples of group actions, Permutation representations; Centralizers and Normalizers, Stabilizers and kernels of group actions; Groups acting on themselves by left multiplication and conjugation with consequences; Cayley's theorem, Conjugacy classes, Class equation, Conjugacy in S_n, Simplicity of A_5.

UNIT – II: Sylow Theorems and Applications (15 hours)

p-groups, Sylow p-subgroups, Sylow's theorem, Applications of Sylow's theorem, Groups of order pq and p^2q (p and q both prime); Finite simple groups, Nonsimplicity tests.

UNIT – III: Solvable Groups and Composition Series (12 hours)

Solvable groups and their properties, Commutator subgroups, Nilpotent groups, Composition series, Jordan-Hölder theorem.

Essential Readings

1. Dummit, David S., & Foote, Richard M. (2004). Abstract Algebra (3rd ed.). John Wiley & Sons. Student Edition, Wiley India 2016.
2. Gallian, Joseph. A. (2017). Contemporary Abstract Algebra (9th ed.). Cengage Learning India Private Limited, Delhi. Indian Reprint 2021.
3. Beachy, John A., & Blair, William D. (2019). Abstract Algebra (4th ed.). Waveland Press.

Suggestive Readings

- Fraleigh, John B., & Brand Neal E. (2021). A First Course in Abstract Algebra (8th ed.). Pearson.
- Herstein, I. N. (1975). Topics in Algebra (2nd ed.). Wiley India. Reprint 2022.
- Rotman, Joseph J. (1995). An Introduction to the Theory of Groups (4th ed.). Springer.

DISCIPLINE SPECIFIC CORE COURSE – 17: ADVANCED LINEAR ALGEBRA

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Advanced Linear Algebra	4	3	1	0	Class XII pass with Mathematics	DSC-4: Linear Algebra

Learning Objectives: The objective of the course is to introduce:
- Linear functionals, dual basis and the dual (or transpose) of a linear transformation.
- Diagonalization problem and Jordan canonical form for linear operators or matrices using eigenvalues.
- Inner product, norm, Cauchy-Schwarz inequality, and orthogonality on real or complex vector spaces.
- The adjoint of a linear operator with application to least squares approximation and minimal solutions to linear system.
- Characterization of self-adjoint (or normal) operators on real (or complex) spaces in terms of orthonormal bases of eigenvectors and their corresponding eigenvalues.

Learning Outcomes: This course will enable the students to:
- Understand the notion of an inner product space in a general setting and how the notion of inner products can be used to define orthogonal vectors, including to the Gram-Schmidt process to generate an orthonormal set of vectors.
- Use eigenvectors and eigenspaces to determine the diagonalizability of a linear operator.
- Find the Jordan canonical form of matrices when they are not diagonalizable.

- Learn about normal, self-adjoint, and unitary operators and their properties, including the spectral decomposition of a linear operator.
- Find the singular value decomposition of a matrix.

SYLLABUS OF DSC-17

UNIT-I: Dual Spaces, Diagonalizable Operators and Canonical Forms (18 hours)

The change of coordinate matrix; Dual spaces, Double dual, Dual basis, Transpose of a linear transformation and its matrix in the dual basis, Annihilators; Eigenvalues, eigenvectors, eigenspaces and the characteristic polynomial of a linear operator; Diagonalizability, Direct sum of subspaces, Invariant subspaces and the Cayley-Hamilton theorem; The Jordan canonical form and the minimal polynomial of a linear operator.

UNIT-II: Inner Product Spaces and the Adjoint of a Linear Operator (12 hours)

Inner products and norms, Orthonormal basis, Gram-Schmidt orthogonalization process, Orthogonal complements, Bessel's inequality; Adjoint of a linear operator with applications to least squares approximation and minimal solutions to systems of linear equations.

UNIT-III: Class of Operators and Their Properties (15 hours)

Normal, self-adjoint, unitary and orthogonal operators and their properties; Orthogonal projections and the spectral theorem; Singular value decomposition for matrices.

Essential Reading

1. Friedberg, Stephen H., Insel, Arnold J., & Spence, Lawrence E. (2019). Linear Algebra (5th ed.). Pearson Education India Reprint.

Suggestive Readings

- Hoffman, Kenneth, & Kunze, Ray Alden (1978). Linear Algebra (2nd ed.). Prentice Hall of India Pvt. Limited. Delhi. Pearson Education India Reprint, 2015.
- Lang, Serge (1987). Linear Algebra (3rd ed.). Springer.

DISCIPLINE SPECIFIC CORE COURSE – 18: COMPLEX ANALYSIS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Complex Analysis	4	3	0	1	Class XII pass with Mathematics	DSC-2 & 11: Real Analysis, Multivariate Calculus

Learning Objectives: The main objective of this course is to:

- Acquaint with the basic ideas of complex analysis.
- Learn complex-valued functions with visualization through relevant practicals.

- Emphasize on Cauchy's theorems, series expansions and calculation of residues.

Learning Outcomes: The accomplishment of the course will enable the students to:

- Grasp the significance of differentiability of complex-valued functions leading to the understanding of Cauchy-Riemann equations.
- Study some elementary functions and evaluate the contour integrals.
- Learn the role of Cauchy-Goursat theorem and the Cauchy integral formula.
- Expand some simple functions as their Taylor and Laurent series, classify the nature of singularities, find residues, and apply Cauchy Residue theorem to evaluate integrals.

SYLLABUS OF DSC-18

UNIT – I: Analytic and Elementary Functions (15 hours)

Functions of a complex variable and mappings, Limits, Theorems on limits, Limits involving the point at infinity, Continuity and differentiation, Cauchy-Riemann equations and examples, Sufficient conditions for differentiability, Analytic functions and their examples; Exponential, logarithmic, and trigonometric functions.

UNIT – II: Complex Integration (15 hours)

Derivatives of functions, Definite integrals of functions; Contours, Contour integrals and examples, Upper bounds for moduli of contour integrals; Antiderivatives; Cauchy-Goursat theorem; Cauchy integral formula and its extension with consequences; Liouville's theorem and the fundamental theorem of algebra.

UNIT – III: Series and Residues (15 hours)

Taylor and Laurent series with examples; Absolute and uniform convergence of power series, Integration, differentiation and uniqueness of power series; Isolated singular points, Residues, Cauchy's residue theorem, Residue at infinity; Types of isolated singular points, Residues at poles and its examples, An application to evaluate definite integrals involving sines and cosines.

Essential Reading

1. Brown, James Ward, & Churchill, Ruel V. (2014). Complex Variables and Applications (9th ed.). McGraw-Hill Education. Indian Reprint.

Suggestive Readings

- Bak, Joseph & Newman, Donald J. (2010). Complex Analysis (3rd ed.). Undergraduate Texts in Mathematics, Springer.
- Mathews, John H., & Howell, Rusell W. (2012). Complex Analysis for Mathematics and Engineering (6th ed.). Jones & Bartlett Learning. Narosa, Delhi. Indian Edition.
- Zills, Dennis G., & Shanahan, Patrick D. (2003). A First Course in Complex Analysis with Applications. Jones & Bartlett Publishers.

Practical (30 hours)- Practical / Lab work to be performed in Computer Lab:

Modeling of the following similar problems using SageMath/Python/Mathematica/Maple/ MATLAB/Maxima/ Scilab etc.

1. Make a geometric plot to show that the *n*th roots of unity are equally spaced points that lie on the unit circle $C_1(0) = \{z : |z| = 1\}$ and form the vertices of a regular polygon with *n* sides, for *n* = 4, 5, 6, 7, 8.
2. Find all the solutions of the equation $z^3 = 8i$ and represent these geometrically.
3. Write parametric equations and make a parametric plot for an ellipse centered at the origin with horizontal major axis of 4 units and vertical minor axis of 2 units. Show the effect of rotation of this ellipse by an angle of $\frac{\pi}{6}$ radians and shifting of the centre from (0,0) to (2,1), by making a parametric plot.
4. Show that the image of the open disk $D_1(-1-i) = \{z : |z+1+i| < 1\}$ under the linear transformation *w* = *f*(*z*) = (3 – 4*i*) *z* + 6 + 2*i* is the open disk:
   $$D_5(-1+3i) = \{w: |w+1-3i| < 5\}.$$
5. Show that the image of the right half-plane Re *z* = *x* > 1 under the linear transformation *w* = (–1 + *i*)*z* – 2 + 3*i* is the half-plane $v > u + 7$, where *u* = Re(*w*), etc. Plot the map.
6. Show that the image of the right half-plane A = {*z*: Re $z \geq \frac{1}{2}$} under the mapping $w = f(z) = \frac{1}{z}$ is the closed disk $\overline{D_1(1)} = \{w: |w-1| \leq 1\}$ in the *w*- plane.
7. Make a plot of the vertical lines *x* = *a*, for $a = -1, -\frac{1}{2}, \frac{1}{2}, 1$ and the horizontal lines *y* = *b*, for $b = -1, -\frac{1}{2}, \frac{1}{2}, 1$. Find the plot of this grid under the mapping $f(z) = \frac{1}{z}$.
8. Find a parametrization of the polygonal path $C = C_1 + C_2 + C_3$ from –1 + *i* to 3 – *i*, where C_1 is the line from: –1 + *i* to –1, C_2 is the line from: –1 to 1 + *i* and C_3 is the line from 1 + *i* to 3 – *i*. Make a plot of this path.
9. Plot the line segment '*L*' joining the point *A* = 0 to $B = 2 + \frac{\pi}{4}i$ and give an exact calculation of $\int_L e^z dz$.
10. Evaluate $\int_C \frac{1}{z-2} dz$, where *C* is the upper semicircle with radius 1 centered at *z* = 2 oriented in a positive direction.
11. Show that $\int_{C_1} z dz = \int_{C_2} z dz = 4 + 2i$, where C_1 is the line segment from –1 – *i* to 3 + *i* and C_2 is the portion of the parabola $x = y^2 + 2y$ joining –1 – *i* to 3 + *i*. Make plots of two contours C_1 and C_2 joining –1 – *i* to 3 + *i* .
12. Use the ML inequality to show that $\left|\int_C \frac{1}{z^2+1} dz\right| \leq \frac{1}{2\sqrt{5}}$, where *C* is the straight-line segment from 2 to 2 + *i*. While solving, represent the distance from the point *z* to the points *i* and – *i*, respectively, i.e., $|z-i|$ and $|z+i|$ on the complex plane $\mathbb{C}$.
13. Find and plot three different Laurent series representations for the function:
    $f(z) = \frac{3}{2+z-z^2}$, involving powers of *z*.
14. Locate the poles of $f(z) = \frac{1}{5z^4+26z^2+5}$ and specify their order.
15. Locate the zeros and poles of $g(z) = \frac{\pi \cot(\pi z)}{z^2}$ and determine their order. Also justify that Res (*g*, 0) = $-\pi^2/3$.

16. Evaluate $\int_{C_1^+(0)} \exp\left(\frac{2}{z}\right) dz$, where $C_1^+(0)$ denotes the circle $\{z: |z| = 1\}$ with positive orientation. Similarly evaluate $\int_{C_1^+(0)} \frac{1}{z^4+z^3-2z^2} dz$.

B.Sc. (Hons) Mathematics, Semester-VI, DSE-Courses

DISCIPLINE SPECIFIC ELECTIVE COURSE – 4(i): MATHEMATICAL FINANCE

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Mathematical Finance	4	3	0	1	Class XII pass with Mathematics	DSC-3, 11, & 15: Probability and Statistics, Multivariate Calculus, & PDE's

Learning Objectives: The main objective of this course is to:

- Introduce the application of mathematics in the financial world.
- Understand some computational and quantitative techniques required for working in the financial markets and actuarial sciences.

Learning Outcomes: The course will enable the students to:

- Know the basics of financial markets and derivatives including options and futures.
- Learn about pricing and hedging of options.
- Learn the Itô's formula and the Black–Scholes model.
- Understand the concepts of trading strategies.

SYLLABUS OF DSE-4(i)

Unit - I: Interest Rates, Bonds and Derivatives (15 hours)

Interest rates, Types of rates, Measuring interest rates, Zero rates, Bond pricing, Forward rates, Duration, Convexity, Exchange-traded markets and Over-the-counter markets, Derivatives, Forward contracts, Futures contracts, Options, Types of traders, Hedging, Speculation, Arbitrage, No Arbitrage principle, Short selling, Forward price for an investment asset.

Unit - II: Properties of Options and the Binomial Model (15 hours)

Types of options, Option positions, Underlying assets, Factors affecting option prices, Bounds for option prices, Put-call parity (in case of non-dividend paying stock only), Early exercise, Trading strategies involving options (except box spreads, calendar spreads and diagonal spreads), Binomial option pricing model, Risk-neutral valuation (for European and American options on assets following binomial tree model).

Unit - III: The Black-Scholes Model and Hedging Parameters (15 hours)
Brownian motion (Wiener Process), Geometric Brownian Motion (GBM), The process for a stock price, Itô's lemma, Lognormal property of stock prices, Distribution of the rate of return, Expected return, Volatility, Estimating volatility from historical data, Derivation of the Black-Scholes-Merton differential equation, Extension of risk-neutral valuation to assets following GBM (without proof), Black−Scholes formulae for European options, Hedging parameters - The Greek letters: Delta, Gamma, Theta, Rho and Vega; Delta hedging, Gamma hedging.

Essential Readings

1. Hull, John C., & Basu, S. (2022). Options, Futures and Other Derivatives (11th ed.). Pearson Education, India.
2. Benninga, S. & Mofkadi, T. (2021). Financial Modeling, (5th ed.). MIT Press, Cambridge, Massachusetts, London, England.

Suggestive Readings

- Luenberger, David G. (2013). Investment Science (2nd ed.). Oxford University Press.
- Ross, Sheldon M. (2011). An elementary Introduction to Mathematical Finance (3rd ed.). Cambridge University Press.
- Day, A.L. (2015). Mastering Financial Mathematics in Microsoft Excel: A Practical Guide for Business Calculations (3rd ed.). Pearson Education Ltd.

Note: Use of non-programmable scientific calculator is allowed in theory examination.

Practical (30 hours)- Practical/Lab work using Excel/R/Python/MATLAB/MATHEMATICA

1. Computing simple, nominal, and effective rates. Conversion and comparison.
2. Computing price and yield of a bond.
3. Comparing spot and forward rates.
4. Computing bond duration and convexity.
5. Trading strategies involving options.
6. Simulating a binomial price path.
7. Computing price of European call and put options when the underlying follows binomial model (using Monte Carlo simulation).
8. Estimating volatility from historical data of stock prices.
9. Simulating lognormal price path.
10. Computing price of European call and put options when the underlying follows lognormal model (using Monte Carlo simulation).

11. Implementing the Black-Scholes formulae.
12. Computing Greeks for European call and put options.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 4(ii): INTEGRAL TRANSFORMS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Integral Transforms	4	3	1	0	Class XII pass with Mathematics	DSC-6,15: ODE's, PDE's DSC-8, 10: Riemann Integration, Sequences & Series of Functions

Learning Objectives: Primary objective of this course is to introduce:

- The basic idea of integral transforms of functions and their applications through an introduction to Fourier series expansion of a periodic function.
- Fourier transform and Laplace transform of functions of a real variable with applications to solve ODE's and PDE's.

Learning Outcomes: The course will enable the students to:

- Understand the Fourier series associated with a periodic function, its convergence, and the Gibbs phenomenon.
- Compute Fourier and Laplace transforms of classes of functions.
- Apply techniques of Fourier and Laplace transforms to solve ordinary and partial differential equations and initial and boundary value problems.

SYLLABUS OF DSE-4(ii)

UNIT-I: Fourier Series and Integrals (18 hours)

Piecewise continuous functions and periodic functions, Systems of orthogonal functions, Fourier series: Convergence, examples and applications of Fourier series, Fourier cosine series and Fourier sine series, The Gibbs phenomenon, Complex Fourier series, Fourier series on an arbitrary interval, The Riemann-Lebesgue lemma, Pointwise convergence, uniform convergence, differentiation, and integration of Fourier series; Fourier integrals.

UNIT-II: Integral Transform Methods (15 hours)

Fourier transforms, Properties of Fourier transforms, Convolution theorem of the Fourier transform, Fourier transforms of step and impulse functions, Fourier sine and cosine transforms, Convolution properties of Fourier transform; Laplace transforms, Properties of Laplace transforms, Convolution theorem and properties of the Laplace transform, Laplace transforms of the heaviside and Dirac delta functions.

UNIT-III: Applications of Integral Transforms (12 hours)

Finite Fourier transforms and applications, Applications of Fourier transform to ordinary and partial differential equations; Applications of Laplace transform to ordinary differential equations, partial differential equations, initial and boundary value problems.

Essential Readings

1. Tyn Myint-U & Lokenath Debnath (2007). Linear Partial Differential Equations for Scientists and Engineers (4th ed.). Birkhauser. Indian Reprint.
2. Lokenath Debnath & Dambaru Bhatta (2015). Integral Transforms and Their Applications (3rd ed.). CRC Press Taylor & Francis Group.

Suggestive Readings

- Baidyanath Patra (2018). An Introduction to Integral Transforms. CRC Press.
- Joel L. Schiff (1999). The Laplace Transform-Theory and Applications. Springer.
- Rajendra Bhatia (2003). Fourier Series (2nd ed.). Texts and Readings in Mathematics, Hindustan Book Agency, Delhi.
- Yitzhak Katznelson (2004). An Introduction to Harmonic Analysis (3rd ed.). Cambridge University Press.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 4(iii): RESEARCH METHODOLOGY

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Research Methodology	4	3	0	1	Class XII pass with Mathematics	NIL

Learning Objectives: The main objective of this course is to:

- Prepare the students with skills needed for successful research in mathematics.
- Develop a basic understanding of how to pursue research in mathematics.
- Prepare students for professions other than teaching, that requires independent mathematical research, critical analysis, and advanced mathematical knowledge.
- Introduce some open source softwares to carry out mathematical research.
- Impart the knowledge of journals, their rankings and the disadvantages of rankings.

Learning Outcomes: The course will enable the students to:

- Develop researchable questions and to make them inquisitive enough to search and verify new mathematical facts.
- Understand the methods in research and carry out independent study in areas of mathematics.
- Write a basic mathematical article and a research project.
- Gain knowledge about publication of research articles in good journals.
- Communicate mathematical ideas both in oral and written forms effectively.

SYLLABUS OF DSE - 4(iii)

UNIT– I: How to Learn, Write, and Research Mathematics (17 hours)

How to learn mathematics, How to write mathematics: Goals of mathematical writing, general principles of mathematical writing, avoiding errors, writing mathematical solutions and proofs, the revision process, What is mathematical research, finding a research topic, Literature survey, Research Criteria, Format of a research article (including examples of mathematical articles) and a research project (report), publishing research.

UNIT- II: Mathematical Typesetting and Presentation using LaTeX (16 hours)

How to present mathematics: Preparing a mathematical talk, Oral presentation, Use of technology which includes LaTeX, PSTricks and Beamer; Poster presentation.

UNIT- III: Mathematical Web Resources and Research Ethics (12 hours)

Web resources- MAA, AMS, SIAM, arXiv, ResearchGate; Journal metrics: Impact factor of journal as per JCR, MCQ, SNIP, SJR, Google Scholar metric; Challenges of journal metrics; Reviews/Databases: MathSciNet, zbMath, Web of Science, Scopus; Ethics with respect to science and research, Plagiarism check using software like URKUND/Ouriginal by Turnitin.

Essential Readings

1. Bindner, Donald, & Erickson Martin (2011). A Student's Guide to the Study, Practice, and Tools of Modern Mathematics. CRC Press, Taylor & Francis Group.
2. Committee on Publication Ethics- COPE (https://publicationethics.org/)
3. Declaration on Research Assessment. https://en.wikipedia.org/wiki/San_Francisco_Declaration_on_Research_Assessment
4. Evaluating Journals using journal metrics; (https://academicguides.waldenu.edu/library/journalmetrics#s-lg-box-13497874)
5. Gallian, Joseph A. (2006). Advice on Giving a Good PowerPoint Presentation (https://www.d.umn.edu/~jgallian/goodPPtalk.pdf). MATH HORIZONS.
6. Lamport, Leslie (2008). LaTeX, a Document Preparation System, Pearson.
7. Locharoenrat, Kitsakorn (2017). Research Methodologies for Beginners, Pan Stanford Publishing Pte. Ltd., Singapore.
8. Nicholas J. Higham. Handbook for writing for the Mathematical Sciences, SIAM, 1998.
9. Steenrod, Norman E., Halmos, Paul R., Schiffer, M. M., & Dieudonné, Jean A. (1973). How to Write Mathematics, American Mathematical Society.

10. Tantau, Till,Wright, Joseph, & Miletić, Vedran (2023). The BEAMER class, Use Guide for Version 3.69. TeX User Group. (https://tug.ctan.org/macros/latex/contrib/beamer/doc/beameruserguide.pdf)
11. University Grants Commission (Promotion of Academic Integrity and Prevention of Plagiarism in Higher Educational Institutions) Regulations 2018 (The Gazette of India: Extraordinary, Part-iii-Sec.4)

Practical (30 hours): Practical work to be performed in the computer lab of the following using any TeX distribution software:

1. Starting LaTeX, Preparing an input file, Sequences and paragraphs, Quotation marks, Dashes, Space after a period, Special symbols, Simple text- generating commands, Emphasizing text, Preventing line breaks, Footnotes, ignorable input.
2. The document, The document class, The title page, Sectioning, Displayed material, Quotations, Lists, Displayed formulas, Declarations.
3. Running LaTeX, Changing the type style, Accents, Symbols, Subscripts and superscripts, Fractions, Roots, Ellipsis.
4. Mathematical Symbols, Greek letters, Calligraphic letters, Log-like functions, Arrays, The array environment, Vertical alignment, Delimiters, Multiline formulas.
5. Putting one thing above another, Over and underlining, Accents, Stacking symbols, Spacing in math mode, Changing style in math mode, Type style, Math style.
6. Defining commands, Defining environments, Theorems.
7. Figure and tables, Marginal notes, The tabbing environment, The tabular environment.
8. The Table and contents, Cross-references, Bibliography and citation.
9. Beamer: Templates, Frames, Title page frame, Blocks, Simple overlays, Themes.
10. PSTricks
11. Demonstration of web resources.

B.A. (Prog.) Semester-VI with Mathematics as Major

Category-II

DISCIPLINE SPECIFIC CORE COURSE (DSC-6): ELEMENTARY MATHEMATICAL ANALYSIS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Elementary Mathematical Analysis	4	3	1	0	Class XII pass with Mathematics	Discipline A-5: Elements of Real Analysis

Learning Objectives: The primary objective of this course is to introduce:

- Sequential criterion for limits and continuity of real-valued functions.
- Riemann integral of real-valued function f on $[a, b]$ using Darboux sums.
- Pointwise and uniform convergence of sequences and series of functions.

Learning Outcomes: This course will enable the students to:

- Apply sequential continuity criterion for the proof of intermediate value theorem.
- Understand the basic tool used to calculate integrals.
- Apply uniform convergence for term-by-term integration in power series expansion.

SYLLABUS OF DSC-6

UNIT-I: Continuous Functions (12 hours)

Sequential criterion for limits and continuity of functions, Continuity on intervals, Intermediate value theorem and applications; Uniform continuity.

UNIT-II: The Riemann Integral (15 hours)

Riemann integration, criterion for integrability and examples; Integrability of continuous and monotone functions, Algebraic properties of the Riemann integral, Fundamental theorem of calculus (first form).

UNIT-III: Uniform Convergence (18 hours)

Sequences and series of functions: Pointwise and uniform convergence, Uniform Cauchy criterion, Weierstrass M-test, Implications of uniform convergence in calculus; Power series, Radius and interval of convergence, Applications of Abel's theorem for power series.

Essential Reading

1. Denlinger, Charles G. (2011). Elements of Real Analysis. Jones & Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

Suggestive Readings

- Bartle, Robert G., & Sherbert, Donald R. (2011). Introduction to Real Analysis (4th ed.). John Wiley & Sons. Wiley India Edition 2015.
- Ross, Kenneth A. (2013). Elementary Analysis: The Theory of Calculus (2nd ed.). Undergraduate Texts in Mathematics, Springer. Indian Reprint.

DISCIPLINE SPECIFIC CORE COURSE – 6 (Discipline A-6): PROBABILITY AND STATISTICS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Probability and Statistics	**4**	**3**	**0**	**1**	**Class XII pass with Mathematics**	**NIL**

Learning Objectives: The primary objective of this course is to:

- Make the students familiar with the basic statistical concepts and tools which are needed to study situations involving uncertainty or randomness.
- Render the students to several examples and exercises that blend their everyday experiences with their scientific interests to form the basis of data science.

Learning Outcomes: This course will enable the students to:

- Understand some basic concepts and terminology-population, sample, descriptive and inferential statistics including stem-and-leaf plots, dotplots, histograms and boxplots.
- Learn about probability density functions and various univariate distributions such as binomial, hypergeometric, negative binomial, Poisson, normal, exponential, and lognormal.
- Understand the remarkable fact that the empirical frequencies of so many natural populations, exhibit bell-shaped (i.e., normal) curves, using the Central Limit Theorem.
- Measure the scale of association between two variables, and to establish a formulation helping to predict one variable in terms of the other, i.e., correlation and linear regression.

SYLLABUS OF DISCIPLINE A-6

UNIT-I: Descriptive Statistics, Probability, and Discrete Probability Distributions (15 hours)

Descriptive statistics: Populations, Samples, Stem-and-leaf displays, Dotplots, Histograms, Qualitative data, Measures of location, Measures of variability, Boxplots; Sample spaces and events, Probability axioms and properties, Conditional probability, Bayes' theorem, and independent events; Discrete random variables & probability distributions, Expected values; Probability distributions: Binomial, geometric, hypergeometric, negative binomial, Poisson, and Poisson distribution as a limit.

UNIT-II: Continuous Probability Distributions **(15 hours)**
Continuous random variables, Probability density functions, Uniform distribution, Cumulative distribution functions and expected values, The normal, exponential, and lognormal distributions.

UNIT-III: Central Limit Theorem and Regression Analysis **(15 hours)**
Sampling distribution and standard error of the sample mean, Central Limit Theorem, and applications; Scatterplot of bivariate data, Regression line using principle of least squares, Estimation using the regression lines; Sample correlation coefficient and properties.

Practical (30 hours): Software labs using Microsoft Excel or any other spreadsheet.

1) Presentation and analysis of data (univariate and bivariate) by frequency tables, descriptive statistics, stem-and-leaf plots, dotplots, histograms, boxplots, comparative boxplots, and probability plots ([1] Section 4.6).
2) Fitting of binomial, Poisson, and normal distributions.
3) Illustrating the Central Limit Theorem through Excel.
4) Fitting of regression line using the principle of least squares.
5) Computation of sample correlation coefficient.

Essential Reading

1. Devore, Jay L. (2016). Probability and Statistics for Engineering and the Sciences (9th ed.). Cengage Learning India Private Limited. Delhi. Indian Reprint 2022.

Suggestive Reading

- Mood, A. M., Graybill, F. A., & Boes, D. C. (1974). Introduction to the Theory of Statistics (3rd ed.). Tata McGraw-Hill Pub. Co. Ltd. Reprinted 2017.

DSE Courses of B.A. (Prog.) Semester-VI
Category-II

DISCIPLINE SPECIFIC ELECTIVE COURSE – 2(i): DISCRETE DYNAMICAL SYSTEMS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		

Discrete Dynamical Systems	**4**	**3**	**0**	**1**	**Class XII pass with Mathematics**	**NIL**

Learning Objectives: The primary objective of this course is to introduce:

- The fundamental concepts of discrete dynamical systems and emphasis on its study through several applications.
- The concepts of the fixed points, chaos and Lyapunov exponents for linear and nonlinear equations have been explained through examples.
- Various applications of chaos in higher dimensional models.

Learning Outcomes: This course will enable the students to:

- Understand the basic concepts of difference equation, chaos and Lyapunov exponents.
- Obtain fixed points and discuss the stability of the dynamical system.
- Find Lyapunov exponents, Bifurcation, and Period-doubling for nonlinear equations.
- Analyze the behavior of different realistic systems with chaos cascade.

SYLLABUS OF DSE-2(i)

UNIT-I: Discrete-time Models (12 hours)

Dynamical systems concepts and examples; Some linear models: Bouncing ball, investment growth, population growth, financial, economic and linear price models; Nonlinear models: Density-dependent population, contagious-disease, economic and nonlinear price models; Some linear systems models: Prey-predator, competing species, overlapping-generations, and economic systems.

UNIT-II: Linear Equations, Systems, their Solutions and Dynamics (18 hours)

Autonomous, non-autonomous linear equations and their solutions, time series graphs; Homogenous, non-homogeneous equations and their solutions with applications; Dynamics of autonomous linear equations, fixed points, stability, and oscillation; Homogeneous, non-homogeneous linear systems and their dynamics, solution space graphs, fixed points, sinks, sources and saddles.

UNIT-III: Nonlinear Equations, their Dynamics and Chaos (15 hours)

Autonomous nonlinear equations and their dynamics: Exact solutions, fixed points, stability; Cobweb graphs and dynamics: Linearization; Periodic points and cycles: 2-cycles, *m*-cycles, and their stability; Parameterized families; Bifurcation of fixed points and period-doubling; Characterizations and indicators of chaos.

Practical (30 hours)- Use of Excel/SageMath/MATHEMATICA/MATLAB/Scilab Software:

1. If Rs. 200 is deposited every 2 weeks into an account paying 6.5% annual interest compounded bi-weekly with an initial zero balance:
   (a) How long will it take before Rs. 10,000/- is in account?
   (b) During this time how much is deposited and how much comes from interest?

(c) Create a time series graph for the bi-weekly account balances for the first 40 weeks of saving scenario.

[1] Computer Project 2.5 pp. 68

2. (a) How much can be borrowed at an annual interest rate of 6% paid quarterly for 5 years in order to have the payments equal Rs. 1000/- every 3 months.
   (b) What is the unpaid balance on this loan after 4 years.
   (c) Create a time series graph for the unpaid balances each quarter for the loan process.
   [1] Computer Project 2.5 pp. 68
3. Four distinct types of dynamics for any autonomous linear equation:
   $x_{n+1} = a\, x_n + b$ for different values of a and b.
   [1] Dynamics of autonomous linear equation, pp. 74
4. Find all fixed points and determine their stability by generating at least the first 100 iterates for various choices of initial values and observing the dynamics
   a. $I_{n+1} = I_n - r\, I_n + s\, I_n\, (1 - I_n\, 10^{-6})$
      for: (i) $r = 0.5, s = 0.25$, (ii) $r = 0.5, s = 1.75$, (iii) $r = 0.5, s = 2.0$.
   b. $P_{n+1} = \frac{1}{P_n} + 0.75\, P_n + c$
      for: $(i)\ c = 0; (ii)\ c = -1; (iii)\ c = -1.25; (iv)\ c = -1.38.$
   c. $x_{n+1} = a\, x_n\, (1 - x_n^2)$
      for: $(i)\ a = 0.5; (ii)\ a = 1.5; (iii)\ a = 2.25;\ (iv)\ a = 2.3.$
   [1] Computer Project 3.2 pp. 110
5. Determine numerically whether a stable cycle exists for the given parameter values, and if so, its period. Perform at least 200 iterations each time and if a cycle is found (approximately), use the product of derivatives to verify its stability.
   a. $P_{n+1} = r\, P_n\, \left(1 - \frac{P_n}{5000}\right)$, for: $(i)\ r = 3.4;\ \ (ii)\ r = 3.5;$
      $(iii)\ r = 3.566;\ (iv)\ r = 3.569;\ (v)\ r = 3.845.$
   b. $P_{n+1} = r\, P_n\, e^{-P_n/1000}$
      for: $(i)\ r = 5;\ (ii)\ r = 10;\ (iii)\ r = 14;\ (iv)\ r = 14.5;\ (v)\ r = 14.75.$
   [1] Computer Project 3.5 pp. 154
6. Find through numerical experimentation the approximate intervals of stability of the (a) 2-cycle; (b) 4-cycle; (c) 8-cycle; (d) 16-cycle; (e) 32-cycle for the following
   a. $f_r(x) = r\, x\, e^{-x}$
   b. $f_r(x) = r\, x^2\, (1 - x)$
   c. $f_a(x) = x\, (a - x^2)$
   d. $f_c(x) = \frac{2}{x} + 0.75\, x - c$
   [1] Computer Project 3.6 pp. 164
7. Through numerical simulation, show that each of the following functions undergoes a period doubling cascade: (**[1] Computer Project 3.7 pp.175**)
   a. $f_r(x) = r\, x\, e^{-x}$
   b. $f_r(x) = r\, x^2\, (1 - x)$
   c. $f_r(x) = r\, x\, e^{-x^2}$
   d. $f_r(x) = \frac{r\, x}{(x^2+1)^2}$
   e. $f_a(x) = x\, (a - x^2)$

8. Discuss (a) Pick two initial points close together, i.e., that perhaps differ by 0.001 or 0.00001, and perform at least 100 iterations of $x_{n+1} = f(x_n)$. Do solutions exhibit sensitive dependence on initial conditions?
   (b) For several random choices of x_0 compute at least 1000 iterates x_n and draw a frequency distribution using at least 50 sub-intervals. Do dense orbits appear to exit?
   (c) Estimate the Lyapunov exponent L by picking several random choices of x_0 and computing $\frac{1}{N}\sum_{n=1}^{N} \ln|f'(x_n)|$ for $N = 1000, 2500, 5000, etc.$
   Does L appear to be positive? i). $f(x) = 2 - x^2$ ii). $f(x) = \frac{2}{x} + \frac{3x}{4} - 2$.
   [1] Computer Project 3.8 pp. 187
9. Show that $f(x) = r\,x\,(1 - x)$ for $r > 4$ and $f(x) = 6.75\,x^2\,(1 - x)$ have horseshoes and homoclinic orbits, and hence chaos. **[1] Computer Project 3.8 pp. 188**
10. Find the fixed point and determine whether it is a sink, source or saddle by iterating and graphing in solution space the first few iterates for several choices of initial conditions. **[1] Computer Project 4.2 pp. 207**
    a. $x_{n+1} = x_n - y_n + 30$
       $y_{n+1} = x_n + y_n - 20.$
    b. $x_{n+1} = x_n + y_n$
       $y_{n+1} = x_n - y_n.$

Essential Reading

1. Marotto, Frederick R. (2006). Introduction to Mathematical Modeling Using Discrete Dynamical Systems. Thomson, Brooks/Cole.

Suggestive Readings

- Devaney, Robert L. (2022). An Introduction to Chaotic Dynamical Systems (3rd ed.). CRC Press, Taylor & Francis Group, LLC.
- Lynch, Stephen (2017). Dynamical Systems with Applications using Mathematica® (2nd ed.). Birkhäuser.
- Martelli, Mario (1999). Introduction to Discrete Dynamical Systems and Chaos. John Wiley & Sons, Inc., New York.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 2(ii): INTRODUCTION TO MATHEMATICAL MODELING

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		

Introduction to Mathematical Modeling	**4**	**3**	**0**	**1**	**Class XII pass with Mathematics**	**Discipline A-3: Differential Equations**

Learning Objectives: The main objective of this course is to introduce:

- Compartmental models and real-life case studies through differential equations, their applications and mathematical modeling.
- Choosing the most appropriate model from competing types that have been fitted.
- Fitting a selected model type or types to the data and making predictions from the collected data.

Learning Outcomes: The course will enable the students to:

- Learn basics of differential equations and compartmental models.
- Formulate differential equations for various mathematical models.
- Construct normal equation of best fit and predict the future values.

SYLLABUS OF DSE-2(ii)

UNIT-I: Compartmental Models (15 hours)

Compartmental diagram and balance law; Exponential decay, radioactive dating, and lake pollution models; Case study: Lake Burley Griffin; Drug assimilation into the blood; Case study: Dull, dizzy or dead; Exponential growth, Density-dependent growth, Equilibrium solutions and stability of logistic equation, Limited growth with harvesting.

UNIT-II: Interacting Population Models and Phase-plane Analysis (15 hours)

SIR model for influenza, Predator-prey model, Ecosystem model of competing species, and model of a battle.

UNIT-III: Analytic methods of model fitting and Simulation (15 hours)

Fitting models to data graphically; Chebyshev approximation criterion, Least-square criterion: Straight line, parabolic, power curve; Transformed least-square fit, Choosing a best model. Monte Carlo simulation modeling**:** Simulating deterministic behavior (area under a curve, volume under a surface); Generating random numbers: middle-square method, linear congruence; Simulating probabilistic behavior.

Essential Readings

1. Barnes, Belinda & Fulford, Glenn R. (2015). Mathematical Modelling with Case Studies, Using Maple and MATLAB (3rd ed.). CRC Press, Taylor & Francis Group.
2. Giordano, Frank R., Fox, William P., & Horton, Steven B. (2014). A First Course in Mathematical Modeling (5th ed.). CENGAGE Learning India.

Suggestive Readings

- Albright, Brian, & Fox, William P. (2020). Mathematical Modeling with Excel (2nd ed.). CRC Press, Taylor & Francis Group.

- Edwards, C. Henry, Penney, David E., & Calvis, David T. (2015). Differential Equations and Boundary Value Problems: Computing and Modeling (5th ed.). Pearson.

Practical (30 hours)- Practical / Lab work to be performed in Computer Lab:

Modeling of the following problems using Mathematica/MATLAB/Maple/Maxima/Scilab etc.

1. Plotting the solution and describe the physical interpretation of the Mathematical Models mentioned below:
   a. Exponential decay and growth model.
   b. Lake pollution model (with constant/seasonal flow and pollution concentration).
   c. Case of single cold pill and a course of cold pills.
   d. Limited growth of population (with and without harvesting).
   e. Predatory-prey model (basic volterra model, with density dependence, effect of DDT, two prey one predator).
   f. Epidemic model of influenza (basic epidemic model, contagious for life, disease with carriers).
   g. Ecosystem model of competing species
   h. Battle model

2. Random number generation and then use it to simulate area under a curve and volume under a surface.
3. Write a computer program that finds the least-squares estimates of the coefficients in the following models.
   a. $y = a\,x^2 + b\,x + c$
   b. $y = a\,x^n$

4. Write a computer program that uses Equations (3.4) in [3] and the appropriate transformed data to estimate the parameters of the following models.
   a. $y = b\,x^n$
   b. $y = b\,e^{a\,x}$
   c. $y = a \ln x + b$
   d. $y = a\,x^2$
   e. $y = a\,x^3$.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 2(iii): RESEARCH METHODOLOGY

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/		

				Practice		
Research Methodology	4	3	0	1	Class XII pass with Mathematics	NIL

Learning Objectives: The main objective of this course is to:

- Prepare the students with skills needed for successful research in mathematics.
- Develop a basic understanding of how to pursue research in mathematics.
- Prepare students for professions other than teaching, that requires independent mathematical research, critical analysis, and advanced mathematical knowledge.
- Introduce some open source softwares to carry out mathematical research.
- Impart the knowledge of journals, their rankings and the disadvantages of rankings.

Learning Outcomes: The course will enable the students to:

- Develop researchable questions and to make them inquisitive enough to search and verify new mathematical facts.
- Understand the methods in research and carry out independent study in areas of mathematics.
- Write a basic mathematical article and a research project.
- Gain knowledge about publication of research articles in good journals.
- Communicate mathematical ideas both in oral and written forms effectively.

SYLLABUS OF DSE - 2(iii)

UNIT– I: How to Learn, Write, and Research Mathematics (17 hours)

How to learn mathematics, How to write mathematics: Goals of mathematical writing, general principles of mathematical writing, avoiding errors, writing mathematical solutions and proofs, the revision process, What is mathematical research, finding a research topic, Literature survey, Research Criteria, Format of a research article (including examples of mathematical articles) and a research project (report), publishing research.

UNIT- II: Mathematical Typesetting and Presentation using LaTeX (16 hours)

How to present mathematics: Preparing a mathematical talk, Oral presentation, Use of technology which includes LaTeX, PSTricks and Beamer; Poster presentation.

UNIT- III: Mathematical Web Resources and Research Ethics (12 hours)

Web resources- MAA, AMS, SIAM, arXiv, ResearchGate; Journal metrics: Impact factor of journal as per JCR, MCQ, SNIP, SJR, Google Scholar metric; Challenges of journal metrics; Reviews/Databases: MathSciNet, zbMath, Web of Science, Scopus; Ethics with respect to science and research, Plagiarism check using software like URKUND/Ouriginal by Turnitin.

Essential Readings

1. Bindner, Donald, & Erickson Martin (2011). A Student's Guide to the Study, Practice, and Tools of Modern Mathematics. CRC Press, Taylor & Francis Group.
2. Committee on Publication Ethics- COPE (https://publicationethics.org/)
3. Declaration on Research Assessment. https://en.wikipedia.org/wiki/San_Francisco_Declaration_on_Research_Assessment
4. Evaluating Journals using journal metrics; (https://academicguides.waldenu.edu/library/journalmetrics#s-lg-box-13497874)
5. Gallian, Joseph A. (2006). Advice on Giving a Good PowerPoint Presentation (https://www.d.umn.edu/~jgallian/goodPPtalk.pdf). MATH HORIZONS.
6. Lamport, Leslie (2008). LaTeX, a Document Preparation System, Pearson.
7. Locharoenrat, Kitsakorn (2017). Research Methodologies for Beginners, Pan Stanford Publishing Pte. Ltd., Singapore.
8. Nicholas J. Higham. Handbook for writing for the Mathematical Sciences, SIAM, 1998.
9. Steenrod, Norman E., Halmos, Paul R., Schiffer, M. M., & Dieudonné, Jean A. (1973). How to Write Mathematics, American Mathematical Society.
10. Tantau, Till,Wright, Joseph, & Miletić, Vedran (2023). The BEAMER class, Use Guide for Version 3.69. TeX User Group. (https://tug.ctan.org/macros/latex/contrib/beamer/doc/beameruserguide.pdf)
11. University Grants Commission (Promotion of Academic Integrity and Prevention of Plagiarism in Higher Educational Institutions) Regulations 2018 (The Gazette of India: Extraordinary, Part-iii-Sec.4)

Practical (30 hours): Practical work to be performed in the computer lab of the following using any TeX distribution software:

1. Starting LaTeX, Preparing an input file, Sequences and paragraphs, Quotation marks, Dashes, Space after a period, Special symbols, Simple text- generating commands, Emphasizing text, Preventing line breaks, Footnotes, ignorable input.
2. The document, The document class, The title page, Sectioning, Displayed material, Quotations, Lists, Displayed formulas, Declarations.
3. Running LaTeX, Changing the type style, Accents, Symbols, Subscripts and superscripts, Fractions, Roots, Ellipsis.
4. Mathematical Symbols, Greek letters, Calligraphic letters, Log-like functions, Arrays, The array environment, Vertical alignment, Delimiters, Multiline formulas.
5. Putting one thing above another, Over and underlining, Accents, Stacking symbols, Spacing in math mode, Changing style in math mode, Type style, Math style.
6. Defining commands, Defining environments, Theorems.
7. Figure and tables, Marginal notes, The tabbing environment, The tabular environment.
8. The Table and contents, Cross-references, Bibliography and citation.
9. Beamer: Templates, Frames, Title page frame, Blocks, Simple overlays, Themes.
10. PSTricks
11. Demonstration of web resources.

B.Sc. (Prog.)/ BA (Prog.) Semester-VI with Mathematics as non-Major

Category-III

DISCIPLINE SPECIFIC CORE COURSE–6 (Discipline A-6): PROBABILITY AND STATISTICS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Probability and Statistics	4	3	0	1	Class XII pass with Mathematics	NIL

Learning Objectives: The primary objective of this course is to:

- Make the students familiar with the basic statistical concepts and tools which are needed to study situations involving uncertainty or randomness.
- Render the students to several examples and exercises that blend their everyday experiences with their scientific interests to form the basis of data science.

Learning Outcomes: This course will enable the students to:

- Understand some basic concepts and terminology-population, sample, descriptive and inferential statistics including stem-and-leaf plots, dotplots, histograms and boxplots.

- Learn about probability density functions and various univariate distributions such as binomial, hypergeometric, negative binomial, Poisson, normal, exponential, and lognormal.
- Understand the remarkable fact that the empirical frequencies of so many natural populations, exhibit bell-shaped (i.e., normal) curves, using the Central Limit Theorem.
- Measure the scale of association between two variables, and to establish a formulation helping to predict one variable in terms of the other, i.e., correlation and linear regression.

SYLLABUS OF DISCIPLINE A-6

UNIT-I: Descriptive Statistics, Probability, and Discrete Probability Distributions (15 hours)
Descriptive statistics: Populations, Samples, Stem-and-leaf displays, Dotplots, Histograms, Qualitative data, Measures of location, Measures of variability, Boxplots; Sample spaces and events, Probability axioms and properties, Conditional probability, Bayes' theorem, and independent events; Discrete random variables & probability distributions, Expected values; Probability distributions: Binomial, geometric, hypergeometric, negative binomial, Poisson, and Poisson distribution as a limit.

UNIT-II: Continuous Probability Distributions (15 hours)
Continuous random variables, Probability density functions, Uniform distribution, Cumulative distribution functions and expected values, The normal, exponential, and lognormal distributions.

UNIT-III: Central Limit Theorem and Regression Analysis (15 hours)
Sampling distribution and standard error of the sample mean, Central Limit Theorem, and applications; Scatterplot of bivariate data, Regression line using principle of least squares, Estimation using the regression lines; Sample correlation coefficient and properties.

Practical (30 hours)

Software labs using Microsoft Excel or any other spreadsheet.

1) Presentation and analysis of data (univariate and bivariate) by frequency tables, descriptive statistics, stem-and-leaf plots, dotplots, histograms, boxplots, comparative boxplots, and probability plots ([1] Section 4.6).
2) Fitting of binomial, Poisson, and normal distributions.
3) Illustrating the Central Limit Theorem through Excel.
4) Fitting of regression line using the principle of least squares.
5) Computation of sample correlation coefficient.

Essential Reading

1. Devore, Jay L. (2016). Probability and Statistics for Engineering and the Sciences (9th ed.). Cengage Learning India Private Limited. Delhi. Indian Reprint 2022.

Suggestive Reading

- Mood, A. M., Graybill, F. A., & Boes, D. C. (1974). Introduction to the Theory of Statistics (3rd ed.). Tata McGraw-Hill Pub. Co. Ltd. Reprinted 2017.

B.Sc. (Physical Sciences/Mathematical Sciences) Semester-VI with Mathematics as one of the Core Discipline

Category-III

DISCIPLINE SPECIFIC CORE COURSE – 6 (Discipline A-6): PROBABILITY AND STATISTICS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Probability and Statistics	4	3	0	1	Class XII pass with Mathematics	NIL

Learning Objectives: The primary objective of this course is to:

- Make the students familiar with the basic statistical concepts and tools which are needed to study situations involving uncertainty or randomness.
- Render the students to several examples and exercises that blend their everyday experiences with their scientific interests to form the basis of data science.

Learning Outcomes: This course will enable the students to:

- Understand some basic concepts and terminology-population, sample, descriptive and inferential statistics including stem-and-leaf plots, dotplots, histograms and boxplots.
- Learn about probability density functions and various univariate distributions such as binomial, hypergeometric, negative binomial, Poisson, normal, exponential, and lognormal.
- Understand the remarkable fact that the empirical frequencies of so many natural populations, exhibit bell-shaped (i.e., normal) curves, using the Central Limit Theorem.
- Measure the scale of association between two variables, and to establish a formulation helping to predict one variable in terms of the other, i.e., correlation and linear regression.

SYLLABUS OF DISCIPLINE A-6

UNIT-I: Descriptive Statistics, Probability, and Discrete Probability Distributions (15 hours)

Descriptive statistics: Populations, Samples, Stem-and-leaf displays, Dotplots, Histograms, Qualitative data, Measures of location, Measures of variability, Boxplots; Sample spaces and events, Probability axioms and properties, Conditional probability, Bayes' theorem, and independent events; Discrete random variables & probability distributions, Expected values; Probability distributions: Binomial, geometric, hypergeometric, negative binomial, Poisson, and Poisson distribution as a limit.

UNIT-II: Continuous Probability Distributions (15 hours)

Continuous random variables, Probability density functions, Uniform distribution, Cumulative distribution functions and expected values, The normal, exponential, and lognormal distributions.

UNIT-III: Central Limit Theorem and Regression Analysis (15 hours)

Sampling distribution and standard error of the sample mean, Central Limit Theorem, and applications; Scatterplot of bivariate data, Regression line using principle of least squares, Estimation using the regression lines; Sample correlation coefficient and properties.

Practical (30 hours)

Software labs using Microsoft Excel or any other spreadsheet.

1) Presentation and analysis of data (univariate and bivariate) by frequency tables, descriptive statistics, stem-and-leaf plots, dotplots, histograms, boxplots, comparative boxplots, and probability plots ([1] Section 4.6).
2) Fitting of binomial, Poisson, and normal distributions.
3) Illustrating the Central Limit Theorem through Excel.
4) Fitting of regression line using the principle of least squares.
5) Computation of sample correlation coefficient.

Essential Reading

1. Devore, Jay L. (2016). Probability and Statistics for Engineering and the Sciences (9th ed.). Cengage Learning India Private Limited. Delhi. Indian Reprint 2022.

Suggestive Reading

- Mood, A. M., Graybill, F. A., & Boes, D. C. (1974). Introduction to the Theory of Statistics (3rd ed.). Tata McGraw-Hill Pub. Co. Ltd. Reprinted 2017.

DSE Courses of B.Sc. (Physical Sciences/Mathematical Sciences) Sem-VI

Category-III

DISCIPLINE SPECIFIC ELECTIVE COURSE – 4(i): ELEMENTARY MATHEMATICAL ANALYSIS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Elementary Mathematical Analysis	4	3	1	0	Class XII pass with Mathematics	Discipline A-5: Elements of Real Analysis

Learning Objectives: The primary objective of this course is to introduce:
- Sequential criterion for limits and continuity of real-valued functions.
- Riemann integral of real-valued function f on [a, b] using Darboux sums.
- Pointwise and uniform convergence of sequences and series of functions.

Learning Outcomes: This course will enable the students to:
- Apply sequential continuity criterion for the proof of intermediate value theorem.
- Understand the basic tool used to calculate integrals
- Apply uniform convergence for term-by-term integration in power series expansion.

SYLLABUS OF DSE-4(i)

UNIT-I: Continuous Functions (12 hours)
Sequential criterion for limits and continuity of functions, Continuity on intervals, Intermediate value theorem and applications; Uniform continuity.

UNIT-II: The Riemann Integral (15 hours)
Riemann integration, criterion for integrability and examples; Integrability of continuous and monotone functions, Algebraic properties of the Riemann integral, Fundamental theorem of calculus (first form).

UNIT-III: Uniform Convergence (18 hours)
Sequences and series of functions: Pointwise and uniform convergence, Uniform Cauchy criterion, Weierstrass M-test, Implications of uniform convergence in calculus; Power series, Radius and interval of convergence, Applications of Abel's theorem for power series.

Essential Reading

1. Denlinger, Charles G. (2011). Elements of Real Analysis. Jones & Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

Suggestive Readings

- Bartle, Robert G., & Sherbert, Donald R. (2011). Introduction to Real Analysis (4th ed.). John Wiley & Sons. Wiley India Edition 2015.
- Ross, Kenneth A. (2013). Elementary Analysis: The Theory of Calculus (2nd ed.). Undergraduate Texts in Mathematics, Springer. Indian Reprint.

DISCIPLINE SPECIFIC ELECTIVE COURSE-4(ii): INTRODUCTION TO MATHEMATICAL MODELING

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Introduction to Mathematical Modeling	4	3	0	1	Class XII pass with Mathematics	Discipline A-3: Differential Equations

Learning Objectives: The main objective of this course is to introduce:
- Compartmental models and real-life case studies through differential equations, their applications and mathematical modeling.
- Choosing the most appropriate model from competing types that have been fitted.
- Fitting a selected model type or types to the data and making predictions from the collected data.

Learning Outcomes: The course will enable the students to:
- Learn basics of differential equations and compartmental models.
- Formulate differential equations for various mathematical models.
- Construct normal equation of best fit and predict the future values.

SYLLABUS OF DSE-4(ii)

UNIT-I: Compartmental Models (15 hours)

Compartmental diagram and balance law; Exponential decay, radioactive dating, and lake pollution models; Case study: Lake Burley Griffin; Drug assimilation into the blood; Case study: Dull, dizzy or dead; Exponential growth, Density-dependent growth, Equilibrium solutions and stability of logistic equation, Limited growth with harvesting.

UNIT-II: Interacting Population Models and Phase-plane Analysis (15 hours)

SIR model for influenza, Predator-prey model, Ecosystem model of competing species, and model of a battle.

UNIT-III: Analytic methods of model fitting and Simulation (15 hours)

Fitting models to data graphically; Chebyshev approximation criterion, Least-square criterion: Straight line, parabolic, power curve; Transformed least-square fit, Choosing a best model. Monte Carlo simulation modeling**:** Simulating deterministic behavior (area under a curve, volume under a surface); Generating random numbers: middle-square method, linear congruence; Simulating probabilistic behavior.

Essential Readings

1. Barnes, Belinda & Fulford, Glenn R. (2015). Mathematical Modelling with Case Studies, Using Maple and MATLAB (3rd ed.). CRC Press, Taylor & Francis Group.
2. Giordano, Frank R., Fox, William P., & Horton, Steven B. (2014). A First Course in Mathematical Modeling (5th ed.). CENGAGE Learning India.

Suggestive Readings

- Albright, Brian, & Fox, William P. (2020). Mathematical Modeling with Excel (2nd ed.). CRC Press, Taylor & Francis Group.
- Edwards, C. Henry, Penney, David E., & Calvis, David T. (2015). Differential Equations and Boundary Value Problems: Computing and Modeling (5th ed.). Pearson.

Practical (30 hours)- Practical / Lab work to be performed in Computer Lab:

Modeling of the following problems using Mathematica/MATLAB/Maple/Maxima/Scilab etc.

1. Plotting the solution and describe the physical interpretation of the Mathematical Models mentioned below:
   a. Exponential decay and growth model.
   b. Lake pollution model (with constant/seasonal flow and pollution concentration).
   c. Case of single cold pill and a course of cold pills.
   d. Limited growth of population (with and without harvesting).
   e. Predatory-prey model (basic volterra model, with density dependence, effect of DDT, two prey one predator).
   f. Epidemic model of influenza (basic epidemic model, contagious for life, disease with carriers).
   g. Ecosystem model of competing species
   h. Battle model

2. Random number generation and then use it to simulate area under a curve and volume under a surface.
3. Write a computer program that finds the least-squares estimates of the coefficients in the following models.
   a. $y = a\,x^2 + b\,x + c$
   b. $y = a\,x^n$

4. Write a computer program that uses Equations (3.4) in [3] and the appropriate transformed data to estimate the parameters of the following models.
   a. $y = b\,x^n$
   b. $y = b\,e^{a\,x}$
   c. $y = a \ln x + b$
   d. $y = a\,x^2$
   e. $y = a\,x^3$.

DISCIPLINE SPECIFIC ELECTIVE COURSE-4(iii): RESEARCH METHODOLOGY

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		
Research Methodology	4	3	0	1	Class XII pass with Mathematics	NIL

Learning Objectives: The main objective of this course is to:
- Prepare the students with skills needed for successful research in mathematics.
- Develop a basic understanding of how to pursue research in mathematics.
- Prepare students for professions other than teaching, that requires independent mathematical research, critical analysis, and advanced mathematical knowledge.
- Introduce some open source softwares to carry out mathematical research.
- Impart the knowledge of journals, their rankings and the disadvantages of rankings.

Learning Outcomes: The course will enable the students to:
- Develop researchable questions and to make them inquisitive enough to search and verify new mathematical facts.
- Understand the methods in research and carry out independent study in areas of mathematics.
- Write a basic mathematical article and a research project.
- Gain knowledge about publication of research articles in good journals.
- Communicate mathematical ideas both in oral and written forms effectively.

SYLLABUS OF DSE - 4(iii)

UNIT– I: How to Learn, Write, and Research Mathematics (17 hours)

How to learn mathematics, How to write mathematics: Goals of mathematical writing, general principles of mathematical writing, avoiding errors, writing mathematical solutions and proofs, the revision process, What is mathematical research, finding a research topic, Literature survey, Research Criteria, Format of a research article (including examples of mathematical articles) and a research project (report), publishing research.

UNIT- II: Mathematical Typesetting and Presentation using LaTeX (16 hours)
How to present mathematics: Preparing a mathematical talk, Oral presentation, Use of technology which includes LaTeX, PSTricks and Beamer; Poster presentation.

UNIT- III: Mathematical Web Resources and Research Ethics (12 hours)
Web resources- MAA, AMS, SIAM, arXiv, ResearchGate; Journal metrics: Impact factor of journal as per JCR, MCQ, SNIP, SJR, Google Scholar metric; Challenges of journal metrics; Reviews/Databases: MathSciNet, zbMath, Web of Science, Scopus; Ethics with respect to science and research, Plagiarism check using software like URKUND/Ouriginal by Turnitin.

Essential Readings

1. Bindner, Donald, & Erickson Martin (2011). A Student's Guide to the Study, Practice, and Tools of Modern Mathematics. CRC Press, Taylor & Francis Group.
2. Committee on Publication Ethics- COPE (https://publicationethics.org/)
3. Declaration on Research Assessment. https://en.wikipedia.org/wiki/San_Francisco_Declaration_on_Research_Assessment
4. Evaluating Journals using journal metrics; (https://academicguides.waldenu.edu/library/journalmetrics#s-lg-box-13497874)
5. Gallian, Joseph A. (2006). Advice on Giving a Good PowerPoint Presentation (https://www.d.umn.edu/~jgallian/goodPPtalk.pdf). MATH HORIZONS.
6. Lamport, Leslie (2008). LaTeX, a Document Preparation System, Pearson.
7. Locharoenrat, Kitsakorn (2017). Research Methodologies for Beginners, Pan Stanford Publishing Pte. Ltd., Singapore.
8. Nicholas J. Higham. Handbook for writing for the Mathematical Sciences, SIAM, 1998.
9. Steenrod, Norman E., Halmos, Paul R., Schiffer, M. M., & Dieudonné, Jean A. (1973). How to Write Mathematics, American Mathematical Society.
10. Tantau, Till,Wright, Joseph, & Miletić, Vedran (2023). The BEAMER class, Use Guide for Version 3.69. TeX User Group. (https://tug.ctan.org/macros/latex/contrib/beamer/doc/beameruserguide.pdf)
11. University Grants Commission (Promotion of Academic Integrity and Prevention of Plagiarism in Higher Educational Institutions) Regulations 2018 (The Gazette of India: Extraordinary, Part-iii-Sec.4)

Practical (30 hours): Practical work to be performed in the computer lab of the following using any TeX distribution software:

1. Starting LaTeX, Preparing an input file, Sequences and paragraphs, Quotation marks, Dashes, Space after a period, Special symbols, Simple text- generating commands, Emphasizing text, Preventing line breaks, Footnotes, ignorable input.

2. The document, The document class, The title page, Sectioning, Displayed material, Quotations, Lists, Displayed formulas, Declarations.
3. Running LaTeX, Changing the type style, Accents, Symbols, Subscripts and superscripts, Fractions, Roots, Ellipsis.
4. Mathematical Symbols, Greek letters, Calligraphic letters, Log-like functions, Arrays, The array environment, Vertical alignment, Delimiters, Multiline formulas.
5. Putting one thing above another, Over and underlining, Accents, Stacking symbols, Spacing in math mode, Changing style in math mode, Type style, Math style.
6. Defining commands, Defining environments, Theorems.
7. Figure and tables, Marginal notes, The tabbing environment, The tabular environment.
8. The Table and contents, Cross-references, Bibliography and citation.
9. Beamer: Templates, Frames, Title page frame, Blocks, Simple overlays, Themes.
10. PSTricks
11. Demonstration of web resources.

COMMON POOL OF GENERIC ELECTIVES (GE) Semester-VI COURSES OFFERED BY DEPARTMENT OF MATHEMATICS

Category-IV

GENERIC ELECTIVES (GE-6(i)): INTRODUCTION TO MATHEMATICAL MODELING

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course
		Lecture	Tutorial	Practical/ Practice		
Introduction to Mathematical Modeling	4	3	0	1	Class XII pass with Mathematics	GE-3(i): Differential Equations

Learning Objectives: The main objective of this course is to introduce:

- Compartmental models and real-life case studies through differential equations, their applications and mathematical modeling.
- Choosing the most appropriate model from competing types that have been fitted.
- Fitting a selected model type or types to the data and making predictions from the collected data.

Learning Outcomes: The course will enable the students to:

- Learn basics of differential equations and compartmental models.
- Formulate differential equations for various mathematical models.
- Construct normal equation of best fit and predict the future values.

SYLLABUS OF GE-6(i)

UNIT-I: Compartmental Models (15 hours)

Compartmental diagram and balance law; Exponential decay, radioactive dating, and lake pollution models; Case study: Lake Burley Griffin; Drug assimilation into the blood; Case study: Dull, dizzy or dead; Exponential growth, Density-dependent growth, Equilibrium solutions and stability of logistic equation, Limited growth with harvesting.

UNIT-II: Interacting Population Models and Phase-plane Analysis (15 hours)

SIR model for influenza, Predator-prey model, Ecosystem model of competing species, and model of a battle.

UNIT-III: Analytic methods of model fitting and Simulation (15 hours)

Fitting models to data graphically; Chebyshev approximation criterion, Least-square criterion: Straight line, parabolic, power curve; Transformed least-square fit, Choosing a best model. Monte Carlo simulation modeling**:** Simulating deterministic behavior (area under a curve, volume under a surface); Generating random numbers: middle-square method, linear congruence; Simulating probabilistic behavior.

Essential Readings

1. Barnes, Belinda & Fulford, Glenn R. (2015). Mathematical Modelling with Case Studies, Using Maple and MATLAB (3rd ed.). CRC Press, Taylor & Francis Group.
2. Giordano, Frank R., Fox, William P., & Horton, Steven B. (2014). A First Course in Mathematical Modeling (5th ed.). CENGAGE Learning India.

Suggestive Readings

- Albright, Brian, & Fox, William P. (2020). Mathematical Modeling with Excel (2nd ed.). CRC Press, Taylor & Francis Group.
- Edwards, C. Henry, Penney, David E., & Calvis, David T. (2015). Differential Equations and Boundary Value Problems: Computing and Modeling (5th ed.). Pearson.

Practical (30 hours)- Practical / Lab work to be performed in Computer Lab: Modeling of the following problems using Mathematica/MATLAB/Maple/Maxima/Scilab etc.

1. Plotting the solution and describe the physical interpretation of the Mathematical Models mentioned below:
   a. Exponential decay and growth model.
   b. Lake pollution model (with constant/seasonal flow and pollution concentration).
   c. Case of single cold pill and a course of cold pills.
   d. Limited growth of population (with and without harvesting).
   e. Predatory-prey model (basic volterra model, with density dependence, effect of DDT, two prey one predator).
   f. Epidemic model of influenza (basic epidemic model, contagious for life, disease with carriers).
   g. Ecosystem model of competing species
   h. Battle model
2. Random number generation and then use it to simulate area under a curve and volume under a surface.
3. Write a computer program that finds the least-squares estimates of the coefficients in the following models.
   a. $y = a\,x^2 + b\,x + c$
   b. $y = a\,x^n$
4. Write a computer program that uses Equations (3.4) in [3] and the appropriate transformed data to estimate the parameters of the following models.
   a. $y = b\,x^n$
   b. $y = b\,e^{a\,x}$
   c. $y = a \ln x + b$
   d. $y = a\,x^2$
   e. $y = a\,x^3$.

GENERIC ELECTIVES (GE-6(ii)): DISCRETE DYNAMICAL SYSTEMS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course
		Lecture	Tutorial	Practical/ Practice		
Discrete Dynamical Systems	4	3	0	1	Class XII pass with Mathematics	NIL

Learning Objectives: The primary objective of this course is to introduce:

- The fundamental concepts of discrete dynamical systems and emphasis on its study through several applications.
- The concepts of the fixed points, chaos and Lyapunov exponents for linear and nonlinear equations have been explained through examples.
- Various applications of chaos in higher dimensional models.

Learning Outcomes: This course will enable the students to:

- Understand the basic concepts of difference equation, chaos and Lyapunov exponents.
- Obtain fixed points and discuss the stability of the dynamical system.
- Find Lyapunov exponents, Bifurcation, and Period-doubling for nonlinear equations.
- Analyze the behavior of different realistic systems with chaos cascade.

SYLLABUS OF GE-6(ii)

UNIT-I: Discrete-time Models (12 hours)

Discrete dynamical systems concepts and examples; Some linear models: Bouncing ball, investment growth, population growth, financial, economic and linear price models; Nonlinear models: Density-dependent population, contagious-disease, economic and nonlinear price models; Some linear systems models: Prey-predator, competing species, overlapping-generations, and economic systems.

UNIT-II: Linear Equations, Systems, their Solutions and Dynamics (18 hours)

Autonomous, non-autonomous linear equations and their solutions, time series graphs; Homogenous, non-homogeneous equations and their solutions with applications; Dynamics of autonomous linear equations, fixed points, stability, and oscillation; Homogeneous, non-homogeneous linear systems and their dynamics, solution space graphs, fixed points, sinks, sources and saddles.

UNIT-III: Nonlinear Equations, their Dynamics and Chaos (15 hours)

Autonomous nonlinear equations and their dynamics: Exact solutions, fixed points, stability; Cobweb graphs and dynamics: Linearization; Periodic points and cycles: 2-cycles, *m*-cycles, and their stability; Parameterized families; Bifurcation of fixed points and period-doubling; Characterizations and indicators of chaos.

Practical (30 hours)- Use of Excel/SageMath/MATHEMATICA/MATLAB/Scilab Software:

1. If Rs. 200 is deposited every 2 weeks into an account paying 6.5% annual interest compounded bi-weekly with an initial zero balance:
   (a) How long will it take before Rs. 10,000/- is in account?
   (b) During this time how much is deposited and how much comes from interest?
   (c) Create a time series graph for the bi-weekly account balances for the first 40 weeks of saving scenario.
   [1] Computer Projects 2.5 pp. 68
2. (a) How much can be borrowed at an annual interest rate of 6% paid quarterly for 5 years in order to have the payments equal Rs. 1000/- every 3 months.
   (b) What is the unpaid balance on this loan after 4 years.
   (c) Create a time series graph for the unpaid balances each quarter for the loan process.
   [1] Computer Projects 2.5 pp. 68
3. Four distinct types of dynamics for any autonomous linear equation:
   $x_{n+1} = a\,x_n + b$ for different values of a and b.
   [1] Dynamics of autonomous linear equation, pp. 74
4. Find all fixed points and determine their stability by generating at least the first 100 iterates for various choices of initial values and observing the dynamics
   a. $I_{n+1} = I_n - r\,I_n + s\,I_n\,(1 - I_n\,10^{-6})$
      for: (i) $r = 0.5, s = 0.25$, (ii) $r = 0.5, s = 1.75$, (iii) $r = 0.5, s = 2.0$.
   b. $P_{n+1} = \frac{1}{P_n} + 0.75\,P_n + c$
      for: $(i)\ c = 0$; $(ii)\ c = -1$; $(iii)\ c = -1.25$; $(iv)\ c = -1.38$.
   c. $x_{n+1} = a\,x_n\,(1 - x_n^2)$
      for: $(i)\ a = 0.5$; $(ii)\ a = 1.5$; $(iii)\ a = 2.25$; $(iv)\ a = 2.3$.
   [1] Computer Projects 3.2 pp. 110
5. Determine numerically whether a stable cycle exists for the given parameter values, and if so, its period. Perform at least 200 iterations each time and if a cycle is found (approximately), use the product of derivatives to verify its stability.
   a. $P_{n+1} = r\,P_n\left(1 - \frac{P_n}{5000}\right)$, for: $(i)\ r = 3.4$; $(ii)\ r = 3.5$;
      $(iii)\ r = 3.566$; $(iv)\ r = 3.569$; $(v)\ r = 3.845$.
   b. $P_{n+1} = r\,P_n\,e^{-P_n/1000}$
      for: $(i)\ r = 5$; $(ii)\ r = 10$; $(iii)\ r = 14$; $(iv)\ r = 14.5$; $(v)\ r = 14.75$.
   [1] Computer Projects 3.5 pp. 154
6. Find through numerical experimentation the approximate intervals of stability of the (a) 2-cycle; (b) 4-cycle; (c) 8-cycle; (d) 16-cycle; (e) 32-cycle for the following
   a. $f_r(x) = r\,x\,e^{-x}$
   b. $f_r(x) = r\,x^2\,(1 - x)$
   c. $f_a(x) = x\,(a - x^2)$
   d. $f_c(x) = \frac{2}{x} + 0.75\,x - c$

[1] Computer Projects 3.6 pp. 164

7. Through numerical simulation, show that each of the following functions undergoes a period doubling cascade:
   a. $f_r(x) = r\,x\,e^{-x}$
   b. $f_r(x) = r\,x^2\,(1-x)$
   c. $f_r(x) = r\,x\,e^{-x^2}$
   d. $f_r(x) = \frac{r\,x}{(x^2+1)^2}$
   e. $f_a(x) = x\,(a - x^2)$

   [1] Computer Projects 3.7 pp. 175
8. Discuss (a) Pick two initial points close together, i.e., that perhaps differ by 0.001 or 0.00001, and perform at least 100 iterations of $x_{n+1} = f(x_n)$. Do solutions exhibit sensitive dependence on initial conditions?
   (b) For several random choices of x_0 compute at least 1000 iterates x_n and draw a frequency distribution using at least 50 sub-intervals. Do dense orbits appear to exit?
   (c) Estimate the Lyapunov exponent L by picking several random choices of x_0 and computing $\frac{1}{N}\sum_{n=1}^{N} \ln|f'(x_n)|$ for $N = 1000, 2500, 5000, etc.$
   Does L appear to be positive? i). $f(x) = 2 - x^2$ ii). $f(x) = \frac{2}{x} + \frac{3x}{4} - 2.$

   [1] Computer Projects 3.8 pp. 187
9. Show that $f(x) = r\,x\,(1-x)$ for $r > 4$ and $f(x) = 6.75\,x^2\,(1-x)$ have horseshoes and homoclinic orbits, and hence chaos. **[1] Computer Projects 3.8 pp. 188**
10. Find the fixed point and determine whether it is a sink, source or saddle by iterating and graphing in solution space the first few iterates for several choices of initial conditions.
    a. $x_{n+1} = x_n - y_n + 30$
       $y_{n+1} = x_n + y_n - 20.$
    b. $x_{n+1} = x_n + y_n$
       $y_{n+1} = x_n - y_n.$

[1] Computer Projects 4.2 pp. 207

Essential Reading

1. Marotto, Frederick R. (2006). Introduction to Mathematical Modeling Using Discrete Dynamical Systems. Thomson, Brooks/Cole.

Suggestive Readings

- Devaney, Robert L. (2022). An Introduction to Chaotic Dynamical Systems (3rd ed.). CRC Press Taylor & Francis Group, LLC.
- Lynch, Stephen (2017). Dynamical Systems with Applications using Mathematica® (2nd ed.). Birkhäuser.
- Martelli, Mario (1999). Introduction to Discrete Dynamical Systems and Chaos. John Wiley & Sons, Inc., New York.